

CLOSURE, UNIFORMITY AND INTENSION: ANOTHER CASE FOR THE ANALYTICITY OF DEDUCTION PREMISES

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ABSTRACT: This paper asks the following question: How is deduction non-ampliative? It answers with a conditional: If deduction is non-ampliative then its premises are non-informative. To establish the truth of this conditional, is the task of this paper. I argue that the non-ampliative nature of deduction entails the non-informative epistemic status of the sentences which are its premises. In other words, by non-informative I do not mean there is no more information in the conclusion than in the premises. This construal of 'non-informative' is already what we mean by 'non-ampliative', and is typically not a controversial claim. By non-informative I mean that the premises themselves are non-informative sentences. Such an account of deduction I call an analytic account of deduction. Three conditions are required to support an analytic account of deduction: 1. epistemic closure, 2. uniformity and 3. intensional semantics. If an analytic account of deduction is correct then it poses problems for philosophical positions that deny the possibility of analytic truth and *a priori* knowledge. But it also poses problems for philosophical positions that accept analytic truth and *a priori* knowledge, but nevertheless maintain that deduction is more informative than a mere rearrangement of meanings or ideas along inferential lines.

KEYWORDS: deduction, analytic truth, non-informative, closure,
uniformity, intensional semantics

Introduction

It seems to me that in addition to this scandal of induction there is an equally disquieting scandal of deduction. Its urgency can be brought home to each of us by any clever freshman who asks, upon being told that deductive reasoning is "tautological" or "analytical" and that logical truths have no "empirical content" and cannot be used to make "factual assertions": In what other sense, then, does deductive reasoning give us new information? Isn't it perfectly obvious that there is some such sense, for what point would there otherwise be to logic and mathematics? This question is apt to cause acute embarrassment, for no such sense has ever been defined in the literature. The only honest answer to our imaginary freshman's question is given by those few unregenerate logical positivists who are bold enough to deny the existence of any objective (non-psychological) sense in which deductive inference yields new information. (Hintikka 1970, 135)

This paper argues that deduction is capable of yielding only analytically true claims and must proceed from only analytically true premises. Such a construal of deduction means it is less useful to philosophers who want it to serve a different purpose in their philosophical work. But, it is argued, for deduction to be able to yield necessary inferences, which is surely its most desirable feature, philosophers have no option but to accept the analyticity of both its premises and its conclusions.

It is not the purpose of this paper to analyse the psycho-socio or even philosophical reasons for why philosophers are so deeply committed to deduction. This task has been well performed by other philosophers, such as Novaes (2021). For an explanation of our evidently deep commitment to deductive inference, I am happy to adopt a weak version of social constructivism about deduction – that it is a tool which we design and then choose to use in order to achieve a specified cognitive end.

I take it that it is of philosophical (not ‘merely’ historical or sociological) significance how a given community of inquirers has come to adopt a particular method of inquiry. Moreover, I maintain that a number of contingent social factors will influence method development and selection. Attending to these social factors is particularly important for methods that have become so influential within a given tradition that one may easily forget that tortuous historical paths and contingencies led to the emergence and establishment of the method in question. (Novaes 2021, 235)

But adopting a social constructivist view of deduction does not imply that the end itself, the body of knowledge which is supposed to be achieved using deduction, is not objective, factual, truth apt and so forth. Deduction appears to comfortably avail itself to practices that aim for objective and factual significance.

It is not by chance that deduction came to occupy such a prominent position in (European) science and mathematics, as it has much to show for itself: it affords a higher degree of certainty for a conclusion (given the truth of the premises) than reasoning methods that do not guarantee truth-preservation [...] (Novaes 2021, 236)

But does it? Does it afford a higher degree of certainty for a conclusion given the truth of the premises? If it does do this, how is this achieved? Why does answering this question matter? If it matters that the means support the ends, then deciding what deduction does, given what philosophy aims to achieve, is important. Those who think deduction is informative have argued their positions in myriad ways, not least of all trying to qualify what is meant by ‘informative’. But all answers to this question are hostage to the particular way in which deduction circumscribes entailment.

[...] there seems to be an inherent tension between the justification and the usefulness of a deduction, given that its very core, necessary truth preservation, is what makes it ultimately non-informative and thus (purportedly) useless. (Novaes 2021, 18)

That deduction is non-ampliative, i.e. that it does not contain *more* information in the conclusion than it has in the premises, is widely accepted as a defining feature of deductive reasoning. This is why we call it tautological. But what the necessary and sufficient conditions are for it being non-ampliative seems to be up for grabs. Such disagreements therefore signal strongly that ‘no more information’ means different things to different theorists. For instance, whether or not deduction can proceed from empirically significant, synthetic or contingently true premises (Devitt 2013; Dennett 2003), can proceed without the need for modal concepts such as necessity and contingency (Studtmann 2010), provides a relationship between concepts and the world which they denote (Hintikka 1970; Genot 2018), can support heavyweight propositions (Hawthorne, 2013) or is informative using truth-functional connectives in natural languages (Kornblith 1994) seem to all be up for grabs.

Consequently, going forward, I’ll use the term ‘informative’ as an umbrella term to describe accounts of deduction that hold that it is in some way more substantive than providing a very narrow scope for rearranging trivially true meaning postulates along inferential lines. If, to some philosophers, rearranged meaning postulates is an epistemically substantive matter, or yields enough information to be called ‘informative’, then all kudos to such philosophers. But, proceeding in good faith asks that we accept that when philosophers argue in defence of the informative nature of deduction, a purely linguistic sense of informative is not what they are after. The camp that argues that deduction is informative typically argues that deduction is more interesting than a rearrangement of trivially true meaning postulates along inferential lines. The arguments to follow are pitted against such positions.

An analytic account of deduction is not new, nor is it unique. It is the position Hume holds about deduction (Stove 1973; Belshaw 1989; Millican 2017) and many philosophers, such as the logical positivists from the Vienna Circle, following Hume, have promulgated this view.

The arguments to follow will draw from the principle of epistemic closure, Hume’s law of uniformity and the theory of intension to develop a case for deduction being analytic.

In the arguments to follow I’ll make use of the following argument, which appears to be a deductive argument, to illustrate key points.

Against-PIT argument

P1 Some source intentional states are extended (i.e. the extended mind thesis is true).

P2 No conscious states are extended (i.e. the extended consciousness thesis is false).

P3 P1 and P2 are mutually inconsistent if PIT¹ is true.

C Therefore, PIT is false. (Turner 2022, 751)

Turner (2022) provides evidence for the truth of both P1 and P2 and also provides evidence for why P1 and P2 are not inconsistent. Given the apparent validity of the argument form, the author takes the truth of the conclusion to have been established by way of entailment. But, since Turner's article does not provide an explicit epistemology of deduction – which is used prolifically throughout the article – I won't force a commitment from Turner about the modal character (i.e. necessary or contingent) of the premises. We therefore proceed with what I take to be a reasonable assumption about the modal character of their, Turner's, premises: Given Turner's positioning of the article within a "cognitive science" (2022, 760), references to scientific credibility (2022, 749 and 759) and further explicit use of physicalist-materialist referents such as "neural mechanisms" (2022, 759), "entities in material environment" (2022, 761), "cases of" (2022, 764), "sensory motor system" (2022, 765) and "unknown whether synthetic consciousness is possible" and "carbon based brain bits" (2022, 767) it is likely that the author takes the premises of the argument above to be informative. Furthermore, the overt commitment to naturalistic philosophy in several places (Turner 2022, 747-750) implies an epistemology which eschews analytic truth and possibly even *a priori* knowledge. But even if such an epistemology is not what Turner subscribes to, their commitment to the contingency of the claims in the Against-PIT argument is enough to use the deduction above as a case in point.

It is possible that both Clark's self-stimulating loop argument and Chalmers' direct access are flawed but that P1 and P2 are nevertheless true, for the truth of the premises might be grounded in reasons that have nothing to do with the existence of self-stimulating feedback loops or the notion that consciousness requires direct access to information for global control (Turner 2022, 770).

¹ PIT refers to 'The Phenomenal Intensional Thesis'. Turner argues that this thesis is false if there are some conscious states that are in fact extended. They (Turner) demonstrates that there are conscious states that are extended. They (Turner) also demonstrates the extended mind thesis and extended consciousness thesis are not inconsistent and is therefore able to draw the conclusion that PIT is false (because PIT assumes these two theses cannot both hold).

I continue as if Against-PIT is a typical case of a 'deductive' argument about which it is assumed the premises are contingently true sentences or propositions.

Going forward, we consider only arguments where the variables and constants of sentences in logical notation have been substituted with natural language terminology like 'and', 'or', 'therefore', 'intentional states', 'conscious states', 'extended', 'electromagnetic field', 'space', 'time', 'Socrates', 'man', 'temperature', 'radiation' and so on. Only natural language deductions present the philosophical problem, i.e. the non-ampliative nature of deduction, I am concerned with here.

I continue with Carnap's (1975) formulation for 'analytic' from *Meaning and Necessity*. This formulation has been chosen because it explicitly yokes analyticity to 'intension', and it strongly implies 'epistemic closure' and 'uniformity' with 'comprehends all possible cases'. If I am to conceptually link 'deduction' with 'analyticity', the best way forward is to show that they share key necessary conditions. So, the sentences which serve as the premises and conclusions of sentences are analytic when they meet the following criteria:

A sentence is analytic in *L* for *X* at *t* if its intension (or range or truth conditions) in *L* for *X* at *t* comprehends all possible cases (Carnap 1975, 243)

Surprisingly, such an analytic account of deduction is not the "standard" one (Stove 1973). It seems that many, if not most, philosophers use deduction informatively, as Turner has. It is a pity that this paper must set forth in defence a non-standard account of deduction, but so be it.

The principles of epistemic closure and uniformity

I continue with the assumption that it is uncontroversial that there cannot be epistemic closure in an empirical system. Any standard endorsement of science will be committed to this claim and any understanding of the average epistemic agent involves an understanding that our sensory systems is not able to know everything there is to know at any given point. We are irrevocably bonded to open epistemic systems for both metaphysical and epistemological reasons. More about this below where I'll provide reason why we are committed to a formulation of epistemic closure which yokes it to a priority.

EC-AP² (epistemic closure with a priority): If one knows *P* and knows with *a priori* certainty that Necessarily $P \supset Q$, one has all that it takes, evidentially speaking, to

² Hawthorne (2013, 42) labels this formulation differently in his own text, but for contextual reasons here, I have labeled it 'EC-AP'.

know Q. (Hawthorne 2013, 42)

I do this even though Dretske and Hawthorne both reject EC-AP. They do this because knowing the sorts of things epistemic agents need to know (e.g. zebras, cookies, fathers, sons, intentional states and extended minds) cannot be known with *a priori* certainty. But they cannot accept EC-AP because they both wed deduction to the world. For Dretske, rational or epistemic agents do not have the capacity to know all possible cases involved in the truth of an entailment. Or as he (Dretske 2013, 28-30) puts it knowing *a posteriori* the truth of the premises of an entailment does not imply knowing *a posteriori* the truth of the conclusion of that entailment. Knowledge is not “transmissible” via entailment (Dretske 2013, 28). We must therefore accept that epistemic agents are unable to demonstrate a necessary entailment in the way deduction implies. His skepticism demands a rejection of deduction. Dretske (2014) argues that, firstly, epistemic closure is a requirement for deduction and, secondly, that since epistemic closure is not possible, deduction isn’t either.

For Hawthorne, however, closure can be construed differently, meaning deduction too can be construed differently, and deductive entailment is therefore possible. For this line of thinking to be plausible, Hawthorne has to provide a formulation of epistemic closure which can proceed without the *a priori*:

EC (epistemic closure): If one knows P and competently deduces Q from P, thereby coming to believe Q, while retaining one’s knowledge that P, one comes to know that Q. (Hawthorne 2013, 42)

Hawthorne has removed the requirement of a priority in EC. This is an improvement if the aim is to be able to offer entailments about empirical reality. Hawthorne has expressed what most theorists, who are not Hintikka’s freshmen, believe about deduction: we use the entailment relationship to transmit our knowledge of the premises to the conclusion.

Hawthorne’s purpose-fit formulation is, unfortunately, untenable. This is because it commits us to an implausible view about our *a posteriori* epistemic capacity and also an implausible view of what can be achieved via entailment. Firstly, we do not know whether the conclusion of an argument is true due to its inclusion in an argument which appears to be deductively valid. And, secondly, if we accept this shortcoming and attribute knowledge of the truth of the conclusion to knowledge of the validity of the entailment we are, in effect, saying we have knowledge of necessary entailments between parts of the world! Even if such entailments were metaphysically actual (note, demonstrating this would take us back to very old school rationalist metaphysics), it would still need to be shown how we could know these (and this would take us back to Kant’s synthetic *a priori*).

If Turner's (2022) Against-PIT argument is in the business of saying something about our mental states, i.e. which are extended and which are not, then it offers more than a rearrangement of meaning postulates along inferential lines. If it were trading only in meaning postulates it would be interested in fixing meaning by stipulation and would avoid doing so with reference to empirical matters of fact. In other words, the truth conditions of the sentences about the extended states, for Turner, lie not in merely saying what we take the terms to mean, but rather in that the propositions expressed are saying something factually true about minds. Turner's Against-PIT argument is embedded in a project seated with naturalistic philosophy trading in contingently true claims. Following Dretske we would say that Against-PIT argues for a necessarily entailed 'heavyweight' proposition about the properties of mind.

Can deductions achieve this? Dretske argues that deduction cannot confer truth, or add to our knowledge, if the knowledge is of heavyweight propositions.

Dretske does not deny that deductive inference often succeeds in adding to our stock of knowledge. But he restricts its knowledge-extending scope: deductive inference adds to our knowledge in just those cases where the deductively reached conclusion is not a "heavyweight" proposition. (Hawthorn 2013, 46)

Despite Dretske's cautionary tale, about deductive inferences being of less substantive type, most philosophers continue as Turner has. This is why the informative account of deduction is the standard one.

The non-standard, analytic account of deduction is the one promulgated by Hume (Hume 1748; Stove 2017; Garrett 2011) the "renegade logical empiricists" (Hintikka 1970, 135) of the opening quote and other philosophers who have remained true to Hume's Fork.

Stove's is a dissenting voice in all of this. He rejects the standard account, and urges instead that we interpret 'demonstrative' as referring to "a (valid) argument from necessarily true premisses" and thus indicates that he believes that Hume's demonstrative arguments form a subset of deductive arguments in general. He claims that only on this reading will Hume be able to argue, as he does repeatedly, that there can be no demonstrative arguments for contingent matters of fact. He wants to find good sense in Hume's account, but suggests that given the standard reading this will not be easy. For that there can be no deductive argument for any matter of fact, any contingent proposition, will strike us as thoroughly implausible. But, Stove insists, the standard reading of 'demonstrative' commits us to just this. (Belshaw 1989, 145)

Here is why Hume's view of demonstrative reasoning provides support for Dretske's first move – that without epistemic closure there can be no deduction. But, unlike Dretske, Hume sensibly does not abandon the *a priori* and analyticity (i.e.

relations of ideas) and, in so doing, is able to offer a theory for how closure is possible and how deduction (or, in Hume's lexicon, 'demonstrative reasoning') works. In short: Hume is not required to reject deduction because he takes it to be proceed from analytically true premises.³ Hume's principle of uniformity, which is mostly used in relation to problems with induction, holds that we are not justified in inferring from experience to non-experience because there is no guarantee that the world will remain as it was from one instance of experience to the next possible instance. There is, of course, disagreement about whether this means Hume rejects induction as genuine inference or not (Garrett and Millican 2011). For current purposes, we only need to take note of the role the principle of uniformity plays in Hume's theory about induction – we do not need to commit to a position about what this means for Hume's view on induction as a legitimate or genuine form of inference. The fact that epistemic agents cannot rely on the world remaining uniform, i.e. that all things do not necessarily remain equal, creates uncertainty for inference. This uncertainty, in the method of science, is cashed out in terms of 'confounding variables'. In epistemology we talk of 'defeaters'. In modal logic, uncertainty is the basis for 'contingency'. When we extrapolate statistically from existing experience that the future will present a particular outcome, we assume that the world will continue to repeat largely the same patterns and present largely the same extraneous variables as are underlying our current experience. Or we assume the variables which are associated with one phenomenon are the same as the variables associated with an inferred phenomenon, if we want the former to entail the latter. In other words, we assume all things remain more or less equal, and then we infer. If we were ill at ease about the world not remaining largely the same across the contexts of the phenomena expressed by the premises and conclusion of an induction, we would not have the confidence to induce. When we infer in this manner, says Hume, we are making unwarranted assumptions about the "constancy" of things (Hume 1748, 100). Constancy is thus epistemically yoked to certainty. The more constant the world is, the greater our certainty when we infer about the world.

³ There are Hume scholars who deny that the analytic account was in fact the one Hume endorsed. Belshaw (1989) discusses this disagreement. For my purposes it is not required to rehearse the details of that disagreement, as it is not my aim to argue for a specific version of Hume. It has been also strongly argued that Hume's claim that only propositions that express "constants" (necessities) can be embedded in demonstrable reasoning is "hopelessly unargued and completely false" (Millican 2017, 21). Nevertheless, I place my unargued stake with the Hume scholars who have equally strongly argued that Hume adopts an analytic account of deduction; scholars such as Stove (1973), Belshaw (1989), and Garrett (2017, 2007), whether they agree with Hume or not.

We all know that the principle of uniformity, which gives rise to Hume's Law (Hare 1954; Erdenck 2015) is used to caution us against inductive leaps.

But, it seems to me, that Hume's Law should be used to caution us with much greater urgency against *necessary* entailments. A lot has been written about Hume's Law and the scandal of induction, but the application of Hume's Law to deduction is patently much more damning. If constancy – or the lack of it – is a cause for concern for inference which is expressly understood and overtly treated as *probabilistic*, then surely the concern it causes is far worse for inference that is taken to be *necessary*? After all, the demand for constancy when one wants to infer necessarily about the world is significantly more exacting. The demand is so exacting it might be impossible to claim such constancy under normal epistemic conditions. This is precisely Dretske's point.

But, unlike Dretske, Hume draws a distinction between reasoning about 'matters of fact' and 'relations of ideas'.

All the objects of human reason or enquiry fall naturally into two kinds, namely relations of ideas and matters of fact. The first kind include geometry, algebra, and arithmetic, and indeed every statement that is either intuitively or demonstratively certain. (Hume 1748, 11)

And for knowledge of the latter we need the *a priori*:

Now whatever is intelligible, and can be distinctively conceived, implies no contradiction, and can never be proved false by any demonstrative argument or abstract reasoning *a priori*. (Hume 1748, 34)

If Hume is right, what would this mean for Turner's argument? How do we explain the appearance of a demonstrated "PIT is false" (Turner 2022, 751)? The appearance of Turner Against-PIT argument being a deduction is because, for Hume, 'demonstrable arguments' are distinct from a claim having been 'demonstrated' (Millican 2017, 24). It is possible for an argument to *appear* as a *demonstration*, but for the conclusion not to have been *demonstrated*. If the premises are taken to be informative or only contingently true (Turner 2022, 770), as in the case of Turner's Against-PIT argument, the conclusion has not been demonstrated – despite appearances. If Hume is right about this then despite the concluding truth seemingly being conferred by the entailment, it is, rather, conferred by further facts. The knowledge has not been transmitted, not necessarily. In other words, the conclusion's truth remains hostage to factors which lie outside of the demonstration itself.

The notion that necessary inference cannot proceed from contingent premises, or from matters of fact, Millican judges a "nonsensical interpretation" on the part of Hume (Millican 2017, 23). Like for so many others, for Millican too, to

limit demonstration or deduction to relations of ideas, or analytically true propositions, is to limit deduction too severely, and is plainly wrong. Others are more sympathetic to Hume's thoughts about demonstration.

So, something that might appear to be a serious oversight, namely a failure to show much concern with deductive arguments whose premises are contingent truths, is in the end a wellfounded omission. For, on top of the rather dubious status of a posteriori knowledge to begin with, there lies a well- founded suspicion that such knowledge cannot be furthered by argument. This, I suggest, is Hume's insight. For while we can construct deductively valid arguments with contingent conclusions for as long as it pleases us, they will suffer under one of two handicaps. Either their premises will not be certain, as when we deductivise inductive arguments by means of some question-begging uniformity principle, or, if the premises are certain, the conclusion of the argument will tell us what we already knew. (Belshaw 1989, 158-159)

The good news, for those who have not eschewed analyticity, is that the constancy that enables necessary entailment or demonstration can be found in closed systems.

The world in which we live might be metaphysically closed or open. If the world is metaphysically open it means at any moment, *t*, the world is subject to change. Newly arising matters of fact are part of its fundamental nature. We would then say the world is not constant or uniform. Metaphysical closedness would mean that the world exists, at time *t*, in its entirety. In such a case the world contains all possible cases at any given moment. We evidently do not know whether the world is an open or closed system. It appears to be open, but that might simply be because worldly matters of fact outrun our epistemic capacity. To *comprehend* all possible cases, therefore, requires *epistemic* closure. But then, if epistemic closure is 'EC' and not 'EC-AP' then it requires an ideal epistemic agent, because it requires an epistemic agent with complete comprehension of all matters of fact, such as which states of mind are extended and which not. This would require complete emersion in the world. Such an agent would require something like 'God's view' (Dummett 2006).

But epistemic agents, as far as we know, are not God-like and presumably for this reason many arguments about closure devolve to positions for and against skepticism (Dretske 2014; Hawthorne 2013; Jope and Pritchard 2023). If epistemic closure were possible for us not God-like epistemic agents, then entailment, whether inductive or deductive, would not be hostage to Hume's Law. In such a world, for such epistemic agents, there might be variables, but they would never be confounding. There would be no defeaters. And there would be no distinct modalities. It is therefore the inadequacy of *a posteriori* epistemic reach about

empirical matters of fact that provides the reason for rejecting Hawthorne's (2013, 42) 'EC' formulation of closure.

For the reasons just rehearsed and given the substantial difficulty that empirical matters of fact import for establishing uniformity and closure, it is surprising that self-proclaimed empiricist philosophers (Quine, 1972; Studtmann 2010; Dennett 2003; Papineau 2016) and philosophers ascribing to naturalistic philosophy, like Turner (2022, 747-750), nevertheless have deduction installed in their arsenal of philosophical tools. Many, if not most, such theorists eschew theoretical categories and cognitive methods other than those that have empirical significance, or can be explained and supported by empirical science. Such philosophers "...generally accept the concept of logical entailment from an empirically discoverable fact" (Studtmann 2010, 27). If, however, it turns out that deduction is used to further arguments about material or scientific reality, but cannot get off the ground under conditions of contingency, then it is difficult to see how to proceed with deduction as an empiricist of this type. It is therefore not a little ironic that, despite the rejection of the analytic-synthetic distinction, Hume remains an exemplar empiricist philosopher to many from this camp:

Nonetheless, the general structure of his anti-metaphysical strategy is sound. Indeed, the present critique so closely parallels Hume's that it may be appropriately called a neo-Humean critique of metaphysics. (Studtmann 2010, 102)

Has Studtmann forgotten that Hume's strategy for denying the legitimacy of metaphysics emerges, precisely, from his Fork. Hume's programme, like for the logical positivists later on, is built on the distinction between relations of ideas and matters of fact, the *a priori* and the *a posteriori*. It is therefore surprising that Studtmann also holds:

Empiricists with respect to knowledge think that all knowledge is ultimately empirical [...] As a result they must make one of two claims about *a priori* knowledge that in the end amount to the same thing: either (1) there is no *a priori* knowledge; or (2) what many have labeled "*a priori* knowledge" is in fact *a posteriori*. (Studtmann 2010, 48-49)

Studtmann (2010) provides an argument for how deduction, or "information decreasing" inference (Studtmann 2010, 28), can work without the *a priori* and without modal concepts like necessity and analyticity. Can his extensional account of deduction explain what happens in Turner's Against-PIT? Can an extensional account provide support for Hawthorn's (2013) 'EC'? Given Studtmann's 1. commitment to all knowledge being *a posteriori* and, 2. his account of deduction free of modal concepts (most importantly, free of analyticity), the standard account of deduction has much invested here.

Studtmann holds that non-ampliative entailment is possible from factual premises and necessity and probability can be disposed of when extensional entailment is used to qualify the entailment relationship. He follows Quine in this regard (Studtmann 2010, 30). Quite simply: For information decreasing entailment, i.e. deductive entailment, extension or reference must be preserved along inferential lines. This means that whatever is being referred to by the sentences which make up the premises, must be the same as what is being referred to in the conclusion. To achieve this is to preserve the truth of the concluding sentence.

Two problems arise for Studtmann: 1. The principle of uniformity and epistemic closure – there is no guarantee of uniformity, and 2. The intension-extension distinction. About the former much has been said already, but here is a further illustration: Even if [P1: Taking Vitamin C prevents getting colds], and [P2: I have taken Vitamin C] are both true, there is no guarantee that necessarily [C: Therefore, I have prevented a cold] is true. And if [C] is true, the argument is not what has established this. This is because there may be no cold viruses in my immediate environment, and therefore what appears as a prevention of a cold is simply the absence of cold viruses. Or, my body might have prevented a cold for other reasons in this instance, even though Vitamin C was taken and/or the reason before. Or, even if P1 and P2 are true, it might be the case that, unwittingly, the Vitamin of P2 I had taken was expired and therefore even having ‘taken’ it I did not really ‘take’ any. It is therefore not the relationship between having taken Vitamin C (in a specific instance or every instance) that is the cause of me not having a cold. There simply is nothing about the logical force of the argument that should convince us of the truth of the conclusion. To know whether not getting a cold is a result of a specific instance of having taken Vitamin C would require clinical corroboration. And even if this turns out to have been two instances that do stand in clinical relation to each other, we do not know this from the force of the argument.

Similarly, if it turns out that any one of Turner’s premises, P1: Some source intentional states are extended (i.e. the extended mind thesis is true), P2: No conscious states are extended (i.e. the extended consciousness thesis is false) and P3: P1 and P2 are mutually inconsistent if PIT is true, turn out to be false, or true for different reasons – which Turner (2022, 770) acknowledges might be possible – the entailment is not valid anymore. Unfortunately, this means that when establishing the truth of the premises is hostage to other, auxiliary and extra-logical acts of truth attribution, validity becomes hostage to the same extra-logical acts of truth attribution. Put another way: We only know if the argument is valid if we know whether the premises and conclusion are true *and* whether the real-world relationship between them holds. If the argument implies causality we must *a*

posteriori establish the causality before we are permitted to say the argument is valid. For contingently true premises, this means we do not know at any point in time whether the deduction we see before us is indeed valid or not – despite appearances.

About the second problem for Studtmann, the intension-extension distinction; Studtmann runs into particular sorts of problems having promulgated an extensional account of logical entailment. Some of these are already discussed under uniformity and closure above. The more direct rejection of an extensional account of deduction is discussed in the next section, when it is shown that only intensional semantics is capable of accommodating uniformity and epistemic closure.

Sterpetti (2019) argues that naturalism cannot provide an account for knowledge of mathematical entities whilst committed solely to knowing *a posteriori* (Sterpetti 2019, 232). He, therefore, argues that naturalising deduction in mathematics requires that mathematics is made non-axiomatic. This allows the problem of the *a priori* to be avoided (Sterpetti 2019, 233, 235, 235) and it avoids a problematic Platonistic ontology of mathematical entities (Sterpetti 2019, 232, 233, 235). Sterpetti does not say how mathematical knowledge would work without axioms, implicitly or explicitly known, but lays out the stakes correctly for the naturalist. The stakes are high when viewing deduction in mathematics as non-axiomatic – this move requires a deeply revisionist stance about mathematics. Fortunately, for my point, even if we could show that mathematical deduction can work without underlying foundational rules or axioms (let's assume it is possible), when deductions are expressed in natural language⁴ the deduction sentences are still subject to the same problems of uniformity and closure. If mathematics proceeds *ad hoc* (axiom-less), then we have an open system. Granted, when we deduce in mathematics we do not overtly take stock of the rules of mathematics. But eliminating the need for explicitly knowing the rules involved in a deductive mathematical operation, does not eliminate the worry about validity being hostage to independently establishing the truth of the premises and conclusion *and* their real-world relationship with each other. With Sterpetti's account, facts might arise that undermine mathematical credibility. The naturalist argument that deduction plays an interdependent role with other forms of reasoning and cognition within a Quinean, defeasible web of beliefs (Devitt 2013; Rey 2014) does not disprove that they are epistemically and modally distinct parts. The interdependency of the

⁴ I am not as concerned with mathematical deduction expressed in numbers and functions as extension and intension collapse for me under these conditions. I assume such mathematical sentences are intensional. My concern is mathematical deduction being used, say, in natural sciences – where the natural language sentences are assumed to refer to [scientific] matters of fact.

different parts of the web of beliefs does not, in itself, dissolve the distinct natures of these parts. My brain and my nervous system are inextricably part of how I perceive the world, but that does not mean we cannot tell them apart.

If not naturalised then how else could deduction be informative? To follow, I argue that we should be equally wary of rationalist (broadly speaking) ways of making deductive informative.

Hintikka (1970) argues that deduction is informative and inferentially valid (non-ampliative). Inference, Hintikka says, is qualified by the 'depth' and 'weight' of a 'unit' (Hintikka 1970, 138). A 'unit' is a singular statement/sentence or a group of statements/sentences, or in this case a deductive proof. The 'weight' of a unit is the contingencies, or "probability-mass" (Hintikka 1970, 138) associated with it. The more contingencies associated with a unit the greater the number of inferred sentences from it. Probability can thus deepen layer upon layer and this is called the 'depth' of a sentence or statement. Inference becomes richer due to the weight of contingencies associated with the propositional content of each premise and the conclusion (Hintikka 1970, 138). The depth of a unit, when part of deductive inference, is called its "surface information", and surface information would be information about "mind-independent reality" (Hintikka 1970, 143-146). To be clear this information is as substantive as "total information we can extract from a sentence concerning the reality to which our language pertains" (Hintikka 1970, 138). But, surface information can be either conceptual or factual. Note, this means that even when conceptual, it is mind-independent. What, however, is not required, for Hintikka, are independent observations, experiments, or factual evidence of any other sort to corroborate the inferred unit, i.e. the conclusion. All we have to do is to look deeper into our own language and engage in an *a priori* analysis of the different contingencies one may encounter in examining the world in which a unit is true (Hintikka 1970, 143). Associated with all purely conceptual truths are empirical facts; facts about the world. There is therefore no reason why the enquirer cannot legitimately take the associated facts as information by only "looking deeper into their language" (Hintikka 1970, 141-145).

For this reason, it may be said that the depth information of a sentence is its surface information after we have subjected it to the whole treatment logic puts at our disposal. Surface information and depth information might thus be called pre-logical and post-logical information, respectively. (Hintikka 1970, 140)

It appears that Hintikka offers conceptual analysis as an epistemology for comprehending the richness of the world that our language denotes. He is committed to the idea that it is impossible to separate, in an *a priori* enquiry, that which is contributed by the 'real world' from that which is purely conceptual. Our

a priori conceptual frameworks are informed by and inform our knowledge of a real and mind-independent reality (Hintikka 1970). If this sounds Kantian, please note that Hintikka does not think “Kant’s formulation is a particularly happy one” (Hintikka 1970, 143). “The deep fact here is that we are relying on the mediation of a certain conceptual system in order to reach reality” (Hintikka 1970, 144). There is nothing wrong, says Hintikka, with trying to get information about the world, unencumbered with *a priori* conceptual systems, but we should remember that to do so is, in the end, impossible. We will always have to start from the conceptual system, the surface, and then discover what is deeper, what is in the world. Deduction, even though conceptual, or maybe because it is conceptual in Hintikka’s sense, is in this way informative. None of this inhibits deductive entailment – because non-ampliative entailment is a function of the conceptual associations moved along inferential lines. By Hintikka’s account there are epistemic relations between concepts and objects, and logical relations between concepts across inferential lines. The logical relations between the concepts justify the epistemic relations between the concepts and the world.

I cannot see how this does not amount to saying that there are logical and necessary relations between parts of the world – the parts expressed in the premises and the parts expressed in the conclusion. Hume, on the contrary, holds that there is no necessary or logical relation between objects (Millican 2017, 23). As Millican (2017, 20-27) explains, for Hume there are two types of supervenience relations: that between objects and that between concepts. Hume argued that only the latter permits logical or necessary demonstration.

Genot (2018) explains that Hintikka sets up a game, based on a logical model of inquiry. He asks us to imagine that the game has two players, the Inquirer and Nature. The Inquirer is allowed to ask Nature questions about what it is like (Genot 2018, 2072). The Inquirer is obliged to ask because it is not locked into a closed epistemic system with Nature. Every bit of additional information gained from the answer to a question will be a variable that needs to be considered when demonstrating the truth of a conclusion about Nature. There are rules for what sorts of questions can be asked. And there are penalties, respectively lesser or greater ‘pay offs’, for asking specific sorts of questions. The more open ended or interrogative the question (wh-questions) the higher the price, whereas whether-questions, i.e. questions that require closed answers of a more definitional or relativised type, do not bear the same costs. When using the Deductive Completeness Theorem, all Nature’s answers add new premises (Genot 2018, 2070-2073).

[I]n the case of pure discovery [...] the choice of the optimal question to be asked is essentially the same as the choice of the optimal premise to draw an inference

from in a purely deductive situation. Thus Sherlock Holmes was right: Strategically speaking, all good reasoning consists of “deductions” if only in the case of pure discovery. (Genot 2018, 2073)

What theorists who are interested in the logic of discovery (new information) are after is to keep the logic of discovery distinct from the logic of justification. The point is that in the logic of discovery, in the formation of the yes-no questions the answers to these questions are not explicitly available within Nature. Such questions would be formulated using “...background knowledge (working memory), while establishing complex chains of conclusiveness conditions involves additional recall of information stored in long-term memory, in particular through associations with the content of perception...” as the premises from which to infer the answers and therefore gain the discovery (Genot 2018, 2079-2081). Genot, following Hintikka, has proposed an account of informative deduction which allows it to provide substantive information within an epistemically closed, but metaphysically open, system. But he left the delineation between logical and more substantive knowledge porous.

Here are two responses: Firstly, has Genot not simply provided a novel account of how the hypothetico-deductive and, to some degree, the deductive-nomological systems work in science? We logically deduce novel hypotheses from existing, epistemically justified hypotheses, but we classify such deductions as *logical* ‘discoveries’ since they are products of logical deduction. In science, however, we regard such deduced hypotheses as up for further ‘epistemic’ justification. In Genot’s game these two justifications collapse. The problem for Genot (and Hintikka) is, therefore, that we still do not have a case wherein the preservation of truth has been secured by the inferential steps. The truth of the conclusion will be preserved only once the system is, so to speak, opened up and epistemic justification can also participate. Hume would say we have an occurrence of a ‘demonstrative reasoning’, but wherein the conclusion has not been ‘demonstrated’. This means, for Hume, necessary entailment has failed.

Secondly, we know that the more premises we add to a deduction the more difficult it becomes to have no danglers and therefore the more difficult it becomes to deduce. Consequently, it is unlikely that logical discovery, whatever that might be, can be achieved when it proceeds from multi-premised demonstrative reasonings. If Turner (2022) had to add every caveat, as a premise, that might undermine the plausibility of Clark’s self-stimulating loop argument and Chalmers’ direct access to the proof, it would become so unwieldy that it is improbable that Turner could write a proof as they have (Turner 2022, 751) with only three premises.

Studtmann, Hintikka and Genot have provided distinct accounts of deduction. It seems that each of them, despite being very different from each other, is consistent with the prevailing standard account of deduction; that it is able to necessarily entail from and to truths about empirical matters of fact. Accounts such as these have been proffered as an antidote to the scandal of deduction. But they aren't. They aren't because they do not de-couple deduction from synthetic truth, and for this reason necessary entailment fails.

Now that we know we need uniformity and epistemic closure for deduction, we need to know how they are achieved.

Intension

In semantics, the distinction between intension and extension is developed as a metatheoretical tool with which to analyse linguistic units, say, sentences. It is a tool which allows us to understand 'Socrates is a man' in two different ways: 1. Extensionally: That the entity which 'Socrates' refers to has the property of being a 'man' – Socrates will have distinguishing features which identify Socrates as a 'man', or 2. Intensionally: That 'Socrates is a man' means that we have a sense, or an idea (if you are Hume), of the historical Greek figure, 'Socrates', as being a man, in that 'man' means 'male sex of the human species'. For the former to be a true sentence, its truth conditions must be instantiated in the world. In other words, there must be objects and properties and classes that are or were the referents of the constituent terms. This makes the sentence an expression of matters of fact. But if a sentence expresses matters of fact, it means that its truth is grounded in an epistemically open system. For an intensionally qualified sentence, however, to be a true its truth conditions obtain when we decide to use the term 'man' as part of what we *mean* by 'Socrates' provided 'Socrates' is also *understood as* 'male historical Greek figure' and is not *understood as*, for instance, 'my mother's dog.' And so on. In this case the truth conditions are dependent only on the relations between the meanings or concepts and to "comprehend[s] all possible cases" (Carnap 1975, 243) is possible because the truth conditions are constructed or stipulated.

Similarly, if 'P2 No conscious states are extended (i.e. the extended consciousness thesis is false)' (Turner 2022, 751) is understood extensionally then its truth conditions are matters of fact – therefore, they may or may not obtain. Presumably extensionally is how Turner qualifies the premise-sentences since he means by 'false' that claims about extended consciousness are substantively false. If Turner had qualified P2 intensionally the additional caveat, 'i.e. the extended consciousness thesis is false', would not be required. This is because the premise itself would be a statement of meaning, in which case all the premise would require for

its truth conditions to be satisfied is that the reader does not understand by the meaning of 'conscious state' something which is 'extended'.

Part of the problem for the much coveted informative nature of deduction arises from the fact that the rules for deduction, the sets of formalised rules that are axiomatic to all different types of deductive logic, are applied to areas of investigation which are not in themselves formalised (Novaes and Reck 2017). This creates a paradox between exactness (formal rules) and fruitfulness (rich language). Formal methods applied to conceptually rich areas of investigation create a tension between what can be formally expressed in natural language and what is required for useful or fruitful concepts in the target area of investigation. This is why even just ambiguity, like the ambiguity of the word 'take' and/or 'taken' in the aforementioned Vitamin C deduction should undermine entailment in an extensional context. Specifically, the tension or paradox is due to the non-formalised areas of investigation being metaphysically and epistemically open. So, when deduction is used in, for instance, natural science it can only be valid if the meanings of the concepts remain exactly the same along inferential lines. This, however, is difficult, if not impossible, when the language is extensional, referential or empirically significant. Here is Arthur Pap on the matter:

More explicitly, the steps of the deduction are: (1) calculation of I for the earth's surface from the measured temperature of the earth and the Stefan-Boltzmann law, (2) calculation of I for the sun's surface from data yielding the sun's (average) distance from the earth and from the inverse square law of heat radiation, (3) calculation of the sun's temperature from the result of (2) and renewed application of the Stefan Boltzmann law. But if the same functor " T_e " has different meanings in the conclusion and in the experimental premise (used in step 1) of the deduction, the deduction cannot be valid. (Pap 1963, 582-583)

The rebuttal to this problem is not as simple as saying, 'Well, we must ensure that we take the meanings of terms to remain the same along referential lines'. Of course, we should! But how do we do that if 'Vitamin C' in the universal claim 'Taking Vitamin C prevents colds' refers to 2000mg Vitamin C and in the particular claim 'I have taken Vitamin C' refers to 50mg Vitamin C? There is nothing in the *explicit* articulation of the deduction which specifies dosage, yet different dosages for the two premises would not undermine the truth of either claim viewed independently. Nevertheless, the dosages being different *should* undermine the validity of the entailment. Only intensionally do we manufacture a language which is simple enough to retain meaning across all premises and the conclusion.

Of course, how to retain such simplicity is also up for grabs. The adoption of intensional semantics has a bearing on how we then think of deduction in terms of the model theoretical vs. proof theoretical dichotomy. Since model theoretical

accounts of deduction want deductive entailment to be qualified as entailment that holds relative to a model such accounts have to assume epistemic closure; models are closed environments, both metaphysically and epistemically. Proof theoretical accounts hold that deductive entailment requires explicitly and overtly demonstrated steps – this amounts to the same demand as made by the Deductive Completeness Theorem. Hume’s sense of ‘demonstrated’, wherein all meaning is contained by the deduction itself, can only be accommodated if the model theoretic *and* proof theoretic demands are satisfied. But the explicit nature of such demonstrative steps is theoretically and practically undermined when the meanings of the terms in the premises and the conclusion are determined extensionally, for reasons much rehearsed already. This means only intensional semantics yokes the truth of the premises to the closed systems advocated in both model and proof theoretic accounts.

Novaes (2021, 73-74) is therefore correct that the model vs proof dichotomy is a false one when trying to understand deduction – precisely because deduction is characterised by both. It is therefore a little surprising that she argues elsewhere that deduction is informative in the sense that it provides a cognitive boost which should be viewed as “closer to fruitfulness than exactness” (Novaes and Reck 2017, 206). If deduction is fruitful, it would fail to meet the requirements needed for model theoretic and proof theoretic systems. Model theoretic accounts of deduction only work when deduction proceeds within a specified and delineated model, like a language, “*L*” (Carnap 1975, 243) – so a closed system. Proof theoretic accounts of deduction only work when conclusions are fully ‘demonstrated’ or ‘proven’. And this cannot occur when the system permits variables, known or unknown. However, when a deduction is cashed out intensionally its truth is constrained by a *L*, for a person or linguistic community, “*X* at a time, *t*” (Carnap 1975, 243). Only under such conditions, conditions of “comprehend[s] all possible cases” (Carnap 1975, 243) *because* there are so few and they are all epistemically available, can all steps be demonstrated. On the other hand, variables are exactly what is needed for fruitfulness (Novaes and Reck 2017, 209). Fruitfulness is therefore at odds with deductive entailment.

One of the stock rebuttals to the analytic account of deduction is that when we deduce and are using empirically significant premises it is implicit in our understanding of deduction that ‘all things remain equal’ from the premises to the conclusion.

Prima facie, the picture is quite attractive. Most deductive extensions of our belief set involve no heavyweight propositions. And in those cases, Dretske allows, the conclusions can be known as long as the premises are too. Though closure fails in full generality, it holds as a *ceteris paribus* generalization. Admittedly, in those

special cases where we do arrive at heavyweight propositions, deductive inference does not extend knowledge. (Hawthorne 2013, 46)

Above, Hawthorne explains Dretske's position: Even when we mandate that 'all things remain equal' along inferential lines, there can be no deductive entailment where propositions are information bearing because then truth is up for grabs on both sides. Therefore, even if no ambiguity, vagueness or equivocation occurs in Turner's deduction – it is still possible that the premises whose truth he defended turn out to be false, with or without Turner or the reader knowing. If it is with their knowledge, then the argument's validity fails. And if without their knowledge, the reader will assume the argument is valid, based on the assumed truth of the premises, but be wrong about the truths and therefore the validity too.

The worry about exactness vs. fruitfulness that Novaes and Reck (2017) raise is echoed by Kornblith (1994) when he hopes to overcome the problems that contingency creates for modus ponens reasoning by distinguishing the truth-functional nature of sentences in logical notation from that of their natural language translations. He explains that, for instance, the natural language concept of 'cause' does not behave truth-functionally speaking, in the way a logical sign for 'implication' behaves. We can "explain [them] away as misunderstandings of the relationship between the surface structure of English and the logical structure of thought" (Kornblith 1994, 251). If Kornblith is right we should regard natural language as fundamentally distinct from logicist language in the manner it confers, transfers and retains truth. This seems correct, except that the standard account of deduction seems to want the benefits of the truth-functional behaviour of logicist languages without foregoing the benefits of the surface structure of, say, English – that its terms are rich in extensional meaning. There is only one way of having both exactness and fruitfulness: Intensional semantics is exactly what permits the surface structure of our language to mirror the logic of thought. This is because it constrains surface structure – meaning – to match exactly that of the structure of thought. Meaning is as such a product of thought, not of the world. Intension does this by tying meaning to the comprehension of "all possible cases" (Carnap 1975, 243) by stipulating truth through legislating how concepts/meanings are related to each other. This is, of course, nothing other than analytic truth. Under conditions of intensionality to say 'Brain states cause thoughts' we may read 'cause' as 'logically implies', and Kornblith's problem is solved. This is because we understand, within the language of the deduction in which the statement appears, that the sentence expresses a conceptual relationship between 'brain states' and 'thought'. Under extensionality we may not conflate 'cause' and 'logically implies' because there

might be cases where there are brain states that do not induce thought, or are only contributory and partial causes for thought.

Arguing in defence of deduction, for example that modus ponens is legitimate (Kornblith 1994), by holding that all we need is “other things being equal” (Kornblith 1994, 257) is not to win the battle for deduction unless, ironically, it is also to submit to the moral that deduction proceeds analytically. “All things being equal” is precisely what we mean by ‘analytic’. As inconvenient as it seems, the only way in which we keep deduction is by so severely constraining the context in which deduction works that necessary entailment would be irreparably and irredeemably ruptured by having contact with emerging information. But the sheer artifice of a context so heavily constrained can only be described as analytic. This demand positions informative accounts of deduction awkwardly: they must advocate to suspend the contingent nature of deductive premises for the deductive entailment to hold – by insisting on ‘all things being equal’, ‘meaning remaining equivalent’ and ‘truth being hypothetical’ and so on. But such a suspension of contingency is exactly what their opponents mean by analyticity. So, the constraints they place give too much to the analytic camp.

Conclusion

I have argued for the truth of this conditional: If deduction is non-ampliative, then its premises are non-informative. This has been established with an appeal to epistemic closure, uniformity and intension. Furthermore, the antecedent of this conditional seems unproblematic and is affirmed by general consensus. Therefore, if the arguments for the truth of the conditional have been successful, we are compelled to bite the bullet with Hume and the logical positivists from Hintikka’s opening quote, and accept the consequent – that its premises are non-informative (or analytic). Deduction is still liberally used to establish myriad philosophical claims – claims that are taken to be substantively true, if true, and are assumed to be informative in a cognitively significant way. We therefore ought to heed accounts, like the one provided here, that prescribe and proscribe how deduction should be viewed and, as a result, appropriately used within philosophical enquiry.

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