

THE CONJUNCTION THESIS FROM NON-AXIOMATIC LOGIC

Miguel López ASTORGA

ABSTRACT: There is a debate over the acceptance or rejection of the conjunction thesis. That debate is linked to the concept of truthmaker. The aim of this paper is not to enter that discussion or try to clarify the concept of truthmaker. The goal is just to address the thesis from Non-Axiomatic Logic. I will attempt to show that the latter logic can be used both accepting and rejecting the conjunction thesis. The key is how the relations between truthmakers and conjunctions are expressed.

KEYWORDS: conjunction thesis, extensional intersection, inheritance copula, non-axiomatic logic, truthmaker

Introduction

The conjunction thesis can be understood in different ways. I will follow Rodríguez-Pereyra's (2009) definition here:

- (1) "If e is a truthmaker for $\langle P \ \& \ Q \rangle$, then e is a truthmaker for $\langle P \rangle$ and e is a truthmaker for $\langle Q \rangle$ " (Rodríguez-Pereyra 2009, 428, italics in text).

The definition includes the metaphysical concept of 'truthmaker'. I will understand it as in Rodríguez-Pereyra (2006), that is, linked to the 'Entailment Principle':

"If a thing makes a certain proposition true and this proposition entails another, then the thing in question also makes the latter proposition true" (Rodríguez-Pereyra 2006, 957).

There is a debate over whether (1) should be accepted (see also, e.g., López de Sa 2009; Yago 2009). I will not enter that debate here. My aim will be to show that there is a logic, Non-Axiomatic Logic, which allows working both assuming (1) and rejecting it. That logic is described in, for example, Wang (2013). As usual in the literature, I will refer to it by the initialism 'NAL'. I will argue that (1) can hold or not in NAL depending on how the relations between truthmakers and conjunctions are expressed. My idea is that the acceptability of (1) in NAL rest on the way we build the inheritance statements (statements with 'inheritance copula', the basic copula in NAL; see also, e.g., Wang 2023) providing those relations.

Given that NAL is a logic enabling to develop a computer system, Non-Axiomatic Reasoning System, a consequence of this paper will be that NARS, which is the initialism of Non-Axiomatic Reasoning System, can operate both adopting the conjunction thesis and excluding it (see also, e.g., Wang 2006, for a description of NARS). To show that, I will need only two components of NAL. The first one is ‘inheritance copula’ (e.g., Wang 2013; Wang 2023). The second component is one of its concepts allowing creating compound terms: ‘extensional intersection’ (e.g., Wang 2013).

The first section of this paper will be devoted to those two components. The second one will explain how we can use those components to incorporate (1). The final section will indicate how those very components can be used to discard the conjunction thesis.

Inheritance copula and extensional intersection

The symbol for inheritance copula in NAL is ‘ $\rightarrow$ ’ (for a description of this copula and its characteristics, see, e.g., Wang 2013). It links two terms. One of them, which is called ‘subject’ and can be expressed by means of symbol S , is the left term. The other one, which is named ‘predicate’ and can be expressed using symbol P , is the right one. An inheritance relation is expressed as (2).

$$(2) \quad “S \rightarrow P” \text{ (e.g., Wang 2023, 5).}$$

But (2) is incomplete. The sense NAL is a non-axiomatic logic is that most of its statements are never absolutely true. This is because there is an essential assumption in NAL: The ‘Assumption of Insufficient Knowledge and Resources’. For my purposes here, one of the consequences of AIKR, which is the initialism of Assumption of Insufficient Knowledge and Resources, is that the system increases or decreases its confidence in a statement as it learns, that is, as it obtains experience (see, e.g., Wang 2011, for further information on AIKR). For this reason, statements have two truth-values in NAL. The first one is the ‘frequency’ value, and the second one is the ‘confidence’ value. Their symbols are, respectively, f and c (e.g., Wang 2013). This is why (2) is incomplete. It should include those values and be expressed as (3).

$$(3) \quad “S \rightarrow P \langle f, c \rangle” \text{ (Wang 2013, 40, Definition 3.8).}$$

To calculate f , formula (4) can be used.

$$(4) \quad “f = w^+ / w” \text{ (Wang 2013, 29, Definition 3.3).}$$

Symbols w^+ and w are, respectively, ‘positive evidence’ and ‘total evidence’ for $S \rightarrow P$. They are calculated resorting to the extension and intension of both S and

P. However, ‘extension’ and ‘intension’ do not have their usual definitions in NAL (Wang 2013). Let us suppose that E is an arbitrary term in the system. Let E^E and E^I be, respectively, the extension of E and its intension. We can use First-Order Predicate Calculus and set theory as metalanguages (which is habitual in NAL, e.g., Wang 2013) to define them. We can claim (see, e.g., Wang 2013, Definition 2.8):

$$\forall x [(x \rightarrow E) \Leftrightarrow (x \in E^E)]$$

Where ‘ $\forall$ ’ represents the universal quantifier, ‘ $\Leftrightarrow$ ’ stands for biconditionality, and ‘ $\in$ ’ is understood as in set theory.

And

$$\forall x [(E \rightarrow x) \Leftrightarrow (x \in E^I)]$$

Given these definitions, w^r and w can be in turn defined:

$$(5) \quad "w^r = |S^E \cap P^E| + |P^I \cap S^I|" \text{ (Wang 2013, 28, Definition 3.2).}$$

Where ‘ $\cap$ ’ keeps its meaning in set theory.

$$(6) \quad "w = |S^E| + |P^I|" \text{ (Wang 2013, 28, Definition 3.2).}$$

Another formula enables to calculate c .

$$(7) \quad "c = w/(w + k)" \text{ (Wang 2013, 29, Definition 3.3).}$$

In (7), k represents a constant. It can have different values. The most habitual one is 1 (e.g., Wang 2013). I will also consider it to be equivalent to 1 here (for the use of ‘ $\rightarrow$ ’, f and c to address other cognitive or logical problems, see, e.g., López-Astorga 2024).

Let us suppose that the system has information on 20 hens. It knows that 12 of those hens are white, and that the other 8 hens are a color different from white. Let us assume that the system does not know anything else about terms *Hen* and *White*. If we resort to formulae (4) and (7), the system can come to (8).

$$(8) \quad Hen \rightarrow White \langle 0.6, 0.95 \rangle$$

An important point for my account below is that, when a statement does not have values assigned, the system attributes default values to it. The default values are 1 and 0.9 for, respectively, f and c (see, e.g., Wang 2013).

NAL also allows the construction of compound terms. That can be done in several ways (e.g., Wang 2013). One of them is by means of the concept of ‘extensional intersection’ (for the use of extensional intersection to address other cognitive or logical problems, see also, e.g., López-Astorga 2025) Let us assume that (9) expresses an extensional intersection between terms E and F .

$$(9) \quad E \Pi F$$

The common symbol in NAL to refer to extensional intersection is symbol ‘ $\cap$ ’, not symbol ‘ Π ’ (e.g., Wang 2013). I will use ‘ Π ’ to represent extensional intersection. That is to distinguish it from intersection as understood in set theory. For the latter intersection, the symbol will be ‘ $\cap$ ’.

What (9) indicates is what (10) expresses.

$$(10) \forall x \{[x \rightarrow (E \Pi F)] \Leftrightarrow [(x \rightarrow E) \wedge (x \rightarrow F)]\} \text{ (see, e.g., Wang 2013, Definition 7.6).}$$

Where ‘ $\wedge$ ’ denotes conjunction.

The extension and intension of a compound term such as (9) can be derived in NAL. Based on Wang (2013, Theorem 7.3), it is possible to state that

$$(11) (E \Pi F)^E = E^E \cap F^E$$

And

$$(12) (E \Pi F)^I = E^I \cup F^I \cup \{(E \Pi F)\}$$

Where ‘ $\cup$ ’ is understood as in set theory.

Let us suppose that the system has compound term (13).

$$(13) \textit{Hen} \Pi \textit{White}$$

It refers to the set of hens that are white. So, its extension is the intersection between the set of hens and the set of things that are white, that is, (14).

$$(14) \textit{Hen}^E \cap \textit{White}^E$$

On the other hand, its intension contains all the characteristics of hens, all the characteristics of things that are white, and compound term (13), that is, (15).

$$(15) \textit{Hen}^I \cup \textit{White}^I \cup \{(\textit{Hen} \Pi \textit{White})\}$$

These components of NAL suffice to show how we can work with the latter logic both accepting (1) and rejecting it. But, before doing that, it is required to make some clarifications.

I have described how inheritance copula and extensional intersection can relate terms. One might think that my description is not pertinent, as (1) refers to truthmakers linking propositions or statements, not terms. However, this is not a problem for my arguments below. NAL can work with ‘high-order statements’ in which the terms are in turn statements. In those cases, inheritance copula is isomorphic to implication and extensional intersection is isomorphic to conjunction (Wang 2013). Accordingly, all the following about inheritance copula and extensional intersection also applies in NAL, respectively, to implication and conjunction.

Accepting the conjunction thesis in NAL

When considering implication between statements, NAL resorts to the usual concepts of ‘sufficient’ and ‘necessary’ conditions (e.g., Wang 2013, Definition 9.3). I will also use them here regarding inheritance copula. The isomorphism between the latter copula and implication justifies that.

If we want (1) to hold in NAL, we need to deem truthmakers as sufficient conditions of conjunctions. If T is a truthmaker for $\langle P \ \& \ Q \rangle$, that can be expressed in NAL as (16).

$$(16) \ T \rightarrow (P \Pi Q) \langle 1, 0.9 \rangle$$

The values for f and c in (16) are the default values. But if T is a truthmaker for $\langle P \ \& \ Q \rangle$, they make sense. The confidence needs to be high, and $c = 0.9$ is a high value. On the other hand, a truthmaker of, for example, G must always cause G to be true, with no exceptions. That means that all the pieces of evidence for the relation between the truthmaker and G should be positive. When that is the case, $w^+ = w$. So, $f = 1$.

According to (1), if (16) holds, (17) and (18) hold, too.

$$(17) \ T \rightarrow P \langle 1, 0.9 \rangle$$

$$(18) \ T \rightarrow Q \langle 1, 0.9 \rangle$$

So, if our intention is to show that (1) is correct in NAL when the relation between T and $\langle P \ \& \ Q \rangle$ is expressed as (16), we need to show that when (16) is correct in NAL, (17) and (18) are correct in NAL as well.

Given that $f = 1$ in (16), (19) is the case.

$$(19) \ T^E \subseteq (P \Pi Q)^E$$

Where ‘ $\subseteq$ ’ works as in set theory.

If (19) is the case, (20) and (21) are also the case.

$$(20) \ T^E \subseteq P^E$$

$$(21) \ T^E \subseteq Q^E$$

Hence, there is no element in T^E that is not in $(P \Pi Q)^E$, P^E , and Q^E at the same time. That implies that all the elements in the extension of T provide positive evidence for (16), (17), and (18). This can be entailed from (5) and (6), which, using T and $(P \Pi Q)$, would be (22) and (23).

$$(22) \ w^+ = |T^E \cap (P \Pi Q)^E| + |(P \Pi Q)^E \cap T^E|$$

$$(23) \ w = |T^E| + |(P \Pi Q)^E|$$

With T and P , they would be (24) and (25).

Miguel López Astorga

$$(24) w^r = |T^E \cap P^E| + |P^I \cap T^I|$$

$$(25) w = |T^E| + |P^I|$$

And resorting to T and Q they would be (26) and (27).

$$(26) w^r = |T^E \cap Q^E| + |Q^I \cap T^I|$$

$$(27) w = |T^E| + |Q^I|$$

Formulae (22) to (27) also include $(P \Pi Q)^I$, P^I , Q^I , and T^I . But they lead to positive evidence, too. If (16) holds, (28) holds as well.

$$(28) (P \Pi Q)^I \subseteq T^I$$

If (28) is correct, (29) is also correct.

$$(29) \{P^I \cup Q^I \cup \{(P \Pi Q)^I\}\} \subseteq T^I$$

Again, based on formulae (22) to (27), we can say that all the possible evidence would be positive. Accordingly, if the fact that T is a truthmaker for $\langle P \& Q \rangle$ is expressed as (16), the conjunction thesis holds in NAL.

Rejecting the conjunction thesis in NAL

If we want (1) not to be correct in NAL, we must consider truthmakers to be necessary conditions of conjunctions. The expression would be now (30) (as in the previous section, in the case of a truthmaker T for $\langle P \& Q \rangle$).

$$(30) (P \Pi Q) \rightarrow T \langle 1, 0.9 \rangle$$

Following (1), if (30) holds, (31) and (32) also hold.

$$(31) P \rightarrow T \langle 1, 0.9 \rangle$$

$$(32) Q \rightarrow T \langle 1, 0.9 \rangle$$

If our intention is to show that (1) is not the case in NAL when we express the relation between T and $\langle P \& Q \rangle$ as (30), we must show that in NAL, when (30) is the case, (31) and (32) are not necessarily the case.

If (30) is correct, (33) is also correct.

$$(33) (P \Pi Q)^E \subseteq T^E$$

On the other hand, for (31) to hold, (34) needs to hold as well.

$$(34) P^E \subseteq T^E$$

But the fact that (33) is the case does not guarantee that (34) is the case. There can be elements in the extension of P that are not in the extension of $(P \Pi Q)$.

Therefore, the possibility exists that not all the pieces of evidence are positive for (31), and that $f \neq 1$, that is, $f < 1$ in (31).

The same happens with (32). The latter statement requires (35) to hold.

$$(35) \quad Q^E \subseteq T^E$$

But, for the same reasons as (34), (35) cannot be derived from (33). There can be elements in the extension of Q that are not in the extension of $(P \Pi Q)$. So, $f \neq 1$, that is, $f < 1$, is also a possible value in (32).

Something similar occurs in the case of intension. (30) implies (36).

$$(36) \quad T' \subseteq (P \Pi Q)'$$

What (36) indicates is (37).

$$(37) \quad T' \subseteq \{P' \cup Q' \cup \{(P \Pi Q)'\}\}$$

If (31) were correct, (38) would have to be correct, too.

$$(38) \quad T' \subseteq P'$$

However, the fact that (37) is the case does not lead to (38). For example, (37) holding, (39) and (40) can also hold.

$$(39) \quad T' \not\subseteq P'$$

$$(40) \quad T' \subseteq Q'$$

Again, the same happens with (32). If (32) is correct, (40) must also be correct. However, (37) does not guarantee that. For example, (37) being the case, (38) and (41) can be the case, too.

$$(41) \quad T' \not\subseteq Q'$$

Therefore, (1) does not hold in NAL if the relation between a truthmaker T and a conjunction $\langle P \ \& \ Q \rangle$ is expressed as (30).

Conclusions

The debate over the conjunction thesis and the metaphysical concept of truthmaker is complex. The purpose of this paper has not been to enter it. I have wanted to show just that we can use a logic such as NAL both if we accept the thesis and if we reject it.

To do that, it is only necessary to consider two components of NAL, which is a logic with a significant set of rules and a rich grammar. This is true to such an extent that a computer system such as NARS can be built from the latter logic. (for some applications of NARS, see, e.g., Fasick 2025; Wang and Awan 2011). Those components are inheritance copula and extensional intersection.

To accept the conjunction thesis in NAL, truthmakers must be deemed as subjects in inheritance statements (or sufficient conditions if we bear in mind the isomorphism between inheritance copula and implication). To rejecting that very thesis, truthmakers must be deemed as predicates in inheritance statements (or necessary conditions if we bear in mind the isomorphism between inheritance copula and implication). As far as conjunctions are concerned, they must be extensional intersections (which are isomorphic to conjunctions) working as predicates in the first case and as subjects in the second one.

If we think about the relation between NAL and NARS, this means that it is possible to build both a computer system recognizing the conjunction thesis and a computer system dismissing that very thesis. It is worth exploring these paths in several senses. Further theoretical analysis of the consequences that can be derived from both alternatives would be relevant. An actual practical implementation of both options would be also interesting.

References

- Fasick, J. 2025. *Non-Axiomatic Reasoning Systems as a substitute for Recommender Systems* [unpublished course project]. Philadelphia: Department of Computer and Information Sciences, College of Science and Technology, Temple University. <https://cis.temple.edu/~wangp/5590-AGI/Projects-2025/FasickJ/Non-Axiomatic%20Reasoning%20Systems%20as%20a%20Substitute%20for%20Recommender%20Systems.pdf>
- López-Astorga, M. 2024. "A non-axiomatic system can deal with apparent nonmonotonicity in the same way as human beings." *Logos & Episteme* XV(4): 463-473. <https://doi.org/10.5840/logos-episteme202415435>
- . 2025. "Computer Chrysippus' conditional and extensional intersection." *Theoria* 91(5): e70033. <https://doi.org/10.1111/theo.70033>
- López de Sa, D. 2009. "Disjunctions, conjunctions, and their truthmakers." *Mind* 118(470): 417-425. <https://doi.org/10.1093/mind/fzp063>
- Rodríguez-Pereyra, G. 2006. "Truthmaking, entailment, and the conjunction thesis." *Mind* 115(460): 957-982. <https://doi.org/10.1093/mind/fzl957>
- . 2009. "The disjunction and conjunction theses." *Mind* 118(470): 427-443. <https://doi.org/10.1093/mind/fzp064>
- Wang, P. 2006. *Rigid Flexibility: The Logic of Intelligence*. Dordrecht: Springer. <https://doi.org/10.1007/1-4020-5045-3>
- . 2011. "The assumptions on knowledge and resources in models of rationality." *International Journal of Machine Consciousness* 3(1): 193-218. <https://doi.org/10.1142/S1793843011000686>

- . 2013. *Non-Axiomatic Logic: A Model of Intelligent Reasoning*. Singapore: World Scientific. <https://doi.org/10.1142/8665>
- . 2023. "The role of copulas in reasoning. Technical Report #17." *Temple AGI Team. Temple University*. <https://cis.temple.edu/tagit/publications/TAGIT-TR-17.pdf>
- Wang, P. & Awan, S. 2011. "Reasoning in Non-Axiomatic Logic: A case study in medical diagnosis." In *Artificial General Intelligence. AGI 2011. Lecture Notes in Computer Science, volume 6830*, edited by J. Schmidhuber, K. R. Thórisson, & M. Looks, 297-302. Heidelberg: Springer, Berlin, Heidelberg. https://doi.org/10.1007/978-3-642-22887-2_33
- Yago, M. 2009. "The conjunction and disjunction theses." *Mind* 118(470): 411-415. <https://doi.org/10.1093/mind/fzp062>