Chisholm's Obsession

Arnold CUSMARIU

ABSTRACT: The characterization problem of mathematical logic yields Chisholm's "particularist" criterion, while the decision problem yields his "methodist" criterion. These connections have not been on anyone's radar screen. To answer the "so what?" requirement in philosophy since Plato, which the criterion-problem literature has failed to do, the article states the problem in argument form and shows that it entails a new kind of skepticism. This clears a path for a way out that no one imagined, Gödel's incompleteness theorem, which can be interpreted as proving that particularist criteria do not entail methodist criteria. This can solve the Cartesian Circle, which is a version of Moore's Paradox. However, I only found pyrite in them thar Hegelian hills that Marxists hike. One outstanding question concerns the impact of these findings on prospects for a unified field theory in physics.

KEYWORDS: The problem of the criterion, Roderick Chisholm, Gödel's Proof, Moore's Paradox, the characterization problem, the decision problem, the Cartesian Circle

1. Prologue

ME

You might be interested to learn that the *Meno* anticipated a problem associated with the second-century philosopher Sextus Empiricus that has come to be called "the problem of the criterion."

SOCRATES

Another footnote?

ME

Yes. Meno asked you this at 80d:

> And in what way will you inquire into something when you do not know what it is at all? Yes, what sort of thing that you do not know will you put forward as an object of inquiry? And even if, at best, you do encounter it, how will you know that this is the thing that you did not know?

SOCRATES

I remember. What of it?

ME

Cave prisoners …

SOCRATES
By Zeus, not again!
ME
Please bear with me. Cave prisoners looking at the shadows passing on the wall in front of them day after day – some of which are shadows of horses and some not – may well be wondering which shadows are shadows of horses and which are not.
SOCRATES
Yes, and…?
ME
The analogy I'm trying to draw is with the point you make in the *Meno* that we should be able to explain, in general, the difference between actions that are virtuous and actions that are not. That is certainly an important question.
SOCRATES
It is.
ME
The form *Virtue* provides the explanation needed. This *aitia* won't help prisoners because they are not acquainted with the form *Horse.* They have no idea this or any other a form even exists. How could they? So, it would not help them to mention your theory of forms. After they leave the Cave and have recovered from a life of illusion, the Forms may help, but not before then.
SOCRATES
That is true.
ME
A general *aitia* of what it is for something to be a horse, even if prisoners had such an *aitia*, by itself would not answer the question how they could tell whether <u>this</u> shadow they see on the wall <u>now</u> is a shadow of a horse and not a shadow of something else. We can't say that it's just a matter of applying a general *aitia* to a specific case because the whole point of the question is what "applying" entails and how the prisoners could be sure their application to <u>this</u> case was correct.
SOCRATES
That is true as well.
ME
But, it seems that the prisoners can know that <u>this</u> shadow is a shadow of a horse only if they already know what a horse is. But then, how can they know what a horse is unless they can already tell if <u>this</u> shadow is a shadow of a horse?
SOCRATES
Interesting question.
ME

Similarly, someone can know that this act is a virtuous act only if he already knows what virtue is. But then, how can he know what virtue is unless he can already tell if this act is a virtuous act?
SOCRATES
My doctrine of recollection solves this problem. We possess knowledge from a previous existence, so all we have to do is recollect that knowledge.
ME
Modern philosophers have not generally accepted this doctrine. They have sought other solutions to the problem of the criterion.
SOCRATES
Did their solutions work?
ME
That is the purpose of this article.

2. Chisholm's Challenge (1973, 1-2)

> "The problem of the criterion" seems to me one of the most important and one of the most difficult problems of philosophy. I am tempted to say that one has not begun to philosophize until one has faced this problem and has recognized how unappealing, in the end, each of the possible solutions is. I have chosen this problem as my topic for the Aquinas Lecture because what first set me to thinking about it (and I remain obsessed by it) were two treatises of twentieth century scholastic philosophy. I refer first to P. Coffey's two volume work *Epistemology or the Theory of Knowledge*, published in 1917. This led me in turn to the treatises of Coffey's great teacher, Cardinal D.J. Mercier: *Critériologie générale ou théorie générale de la certitude*.[1]

3. Problem Sourcing

For a statement of the problem of the criterion, Chisholm quoted in his Aquinas Lecture a passage, not from Sextus Empiricus or *Meno*, but from Montaigne's *Essays*, published in 1588. Here is Sextus Empiricus first, followed by Montaigne.[2]

> Those who claim for themselves to judge the truth are bound to possess a criterion of truth. This criterion, then, either is without a judge's approval or has been approved. But if it is without approval, whence comes it that it is trustworthy? For no matter of dispute is to be trusted without judging. And, if it has been approved,

---

[1] Chisholm used "criterion" in a different sense in his book *Perceiving: A Philosophical Study*: "A mark or criterion, for any subject S, that S has adequate evidence for a given proposition or hypothesis *h*, would be some state or condition of S ..." (Chisholm 1957, 34).
[2] See Chisholm (1973, 3). Chisholm provides the original French text in a footnote, of which his quotation is a paraphrase.

> that which approves it, in turn, either has been approved or has not been approved, and so on ad infinitum.
>
> To know whether things really are as they seem to be, we must have a *procedure* for distinguishing appearances that are true from appearances that are false. But to know whether our procedure is a good procedure, we have to know whether it really *succeeds* in distinguishing appearances that are true from appearances that are false. And we cannot know whether it does really succeed unless we already know which appearances are *true* and which ones are false. And so we are caught in a circle.

I became acquainted with the problem of the criterion from the first philosophy book I ever read, *Socratic Method and Critical Philosophy* by the German philosopher Leonard Nelson (1882-1927).[3] I happened on this book entirely by chance in college – one of many strokes of good fortune in my life – found its contents vaguely familiar (anamnesis?), and decided to change majors from mathematics to philosophy. What got my attention was this passage in Chapter VII, bearing the ominous title "The Impossibility of a Theory of Knowledge" :

> I wish to apply [the Socratic] method to the problem of "the theory of knowledge," also called the problem of knowledge. This problem is that of the objective validity of our knowledge. It is the task of a "theory of knowledge" to test the truth or objective validity of our knowledge. I maintain that a solution of this problem is *impossible*, and I prove this as follows:
>
> In order to solve this problem, we should have to have a criterion by the application of which we could decide whether or not a cognition is true; I shall call it briefly the "validity criterion." This criterion would itself either be or not be a cognition. If it be a cognition, it would fall within the area of what is problematic, the validity of which is first to be solved with the aid of our criterion. Accordingly, it cannot itself be a cognition. But if the criterion be not a cognition, it would nevertheless, in order to be applicable, have to be known, i.e., we should have to know that it is a criterion of the truth. But in order to gain this knowledge of the criterion, we should already have to apply it. In both cases, therefore, we encounter a contradiction. A "validity criterion" is consequently impossible, and hence there can be no "theory of knowledge." (Nelson 1965, 188-89)

More recently, James Van Cleve stated the problem of the criterion this way:

> The problem of the criterion, as it has come to be understood since Chisholm, is

[3] See Nelson 1965. This book is based on lectures Nelson delivered at Göttingen 1909-1927. Chapter VII is reprinted in Chisholm & Swartz 1973, 3-19. Nelson's book is not cited in the bibliography of Amico 1993, the only book-length treatment of the problem of the criterion. Amico's book came into my possession after I completed this article. Similarities are coincidental.

> often understood thus: Do we start with (a) knowledge of general epistemic principles or "criteria," such as principles affirming the reliability of certain sources, or do we start with (b) particular bits of knowledge delivered by these sources, or do we start with neither of these things until the other is in hand. If we start with (a), we may be able to get (b), and if we start with (b), we may be able to get (a), but if we start with neither, we get nowhere.[4] (Van Cleve 2011, 24)

I propose to work with Chisholm's statement of the problem of the criterion. Whether my analysis applies to other version of the problem is another story.

4. Say What?

Implying, suggesting, or even so much as hinting that philosophers who showed no interest in the problem of the criterion had not yet "begun to philosophize" takes a swipe at big names in the history of philosophy – Kant, Quine and Sellars come to mind – though this apparently has not raised eyebrows, not in print as far as I know.

- In hindsight, I wish I'd asked Chisholm to explain this comment.[5]
- I took several of his seminars at Brown University in the 1970s.
- He was one of my PhD dissertation advisers and presided at my oral defense.

5. Puzzling Facts

Chisholm's "obsession" comment is hard for me to explain.

(1) None of the 30 articles in Hahn 1997, dealt with Chisholm's work on the problem of the criterion. The same is true of the 16 chapters of Lehrer 1975; the 15 chapters of Sosa 1979; and Feldman & Feldman 2024.

(2) Chisholm never talked about the problem of the criterion in any of his seminars I attended during 1972-77.

(3) The only "obsession" I remember, if that is the right word, is with the Gettier Problem.[6]

(4) Russell 1945, a book of almost 900 pages, mentions Sextus Empiricus several times (on pages 215, 238 and 265) and Montaigne once (on page 516) but not in connection with the problem of the criterion; the problem itself is not mentioned.

---

[4] The Chisholm reference is to the Aquinas Lecture. Van Cleve assumes (as have others) that criteria and general epistemic principles are equivalent, an assumption I reject. As we shall see, more than one criterion is involved and neither need involve epistemic principles.

[5] Chisholm died in 1999. RIP.

[6] I discuss the Gettier Problem in Ch. 23 of Cusmariu 2026.

6. Back to Chisholm

As if to confirms he was still "obsessed" with the problem of the criterion, Chisholm returned to it in his 1989 edition of *Theory of Knowledge*. There, he set up the problem as he had in his Aquinas Lecture by distinguishing "two general questions" (Chisholm 1989, 6, original italics):[7]

1. *What* do we know?

2. How are we to decide, in any particular case, *whether* we know?

He added,

> The first of these may also be put by asking, "What is the *extent* of our knowledge?" and the second by asking, "What are the *criteria* of knowing?"

Finally, he stated what he understood to be the problem of the criterion this way:

> If we know the answer to either of these questions, then we may devise a procedure that will enable us to answer the other. If we can specify criteria of knowledge, we may have a way of deciding how far our knowledge extends. Or if we know how far our knowledge does extend and are able to say what the things are that we know, then we may be able to formulate criteria enabling us to mark off the things we do know from the things that we do not.
>
> But if we do not have the answer to the first question, then, it would seem, we have no way of answering the second. And if we do not have the answer to the second, then, it would seem, we have no way of answering the first.
>
> Is there a way out?[8]

7. Preliminaries

Contraposition on the two sentences of the last paragraph above, paraphrasing, and making the modal aspect of the problem of the criterion explicit, yields:

A. To decide, in a particular case, *whether* we know, it is necessary to have already determined in general *what* we know.

B. To determine in general *what* we know, it is necessary to have already decided,

---

[7] The 1966 and 1977 editions of *Theory of Knowledge* did not discuss the problem of the criterion. Note that the 1977 edition was published after Chisholm's 1973 Aquinas Lecture.

[8] See McCain for an informative article surveying the problem of the criterion and solutions to it. McCain writes: "It is primarily due to the work of Roderick Chisholm that the Problem of the Criterion is discussed by contemporary epistemologists at all." McCain's bibliography contains some fifty entries. Kant, Quine and Sellars are not on the list. Hegel is on the list but not Wittgenstein.

in a particular case, *whether* we know.

A and B suggest two criteria, C1 and C2:

- Criterion C1: By means of which to determine which propositions in general have the property *being known* and which propositions do not have the property *being known*.
- Criterion C2: By means of which to decide whether or not a given proposition has the property *being known*.
- Claim A says that C2 can be carried out only if C1 has already been carried out.
- Claim B says that C1 can be carried out only if C2 has already been carried out.

8. Key Questions

1. "It is necessary that" is stronger than the material conditional "only if." Does that matter?
2. How exactly do C1 and C2 differ?
3. Can one be defined in terms of the other?
4. Is there a C1 for any C2 and vice-versa?
5. Can the same criterion be used to answer both questions 1 and 2?
6. If so, how exactly would it accomplish that?
7. If not, why not?
8. Is the problem of the criterion a problem only for epistemology?
9. Is the problem of the criterion a problem only for philosophy?
10. How else can the problem be raised and to what effect?
11. Sextus, Montaigne, Nelson and Chisholm agree that circularity of some kind is at work, but of what kind is it and is there anything wrong with it?
12. Is the problem of the criterion an argument and not just an intriguing puzzle?
13. If so, what are the premises and conclusion of the argument?
14. Can the argument be generalized; if so, what problems does it raise elsewhere?
15. A "way out" for arguments means showing they are invalid or unsound. What do Chisholm and the literature on the problem of the criterion say on the matter?

The contributions Descartes and Hegel are discussed below. Wittgenstein's[9] use of "criterion"[10] is too complicated an issue to tackle here.

9. Problem Generalization

We can answer questions (h) and (i) right away. The problem of the criterion can be generalized to show that it is definitely <u>not</u> a problem only for epistemology or only for philosophy.

Let *F* be a property of interest in a theory such that some objects of the theory have *F* and some do not.

For example, *F* can the syntactic property *being well-formed*; or the semantic property *being true*; or the modal property *being contingent*; or the aesthetic property *being beautiful*[11]; or the epistemic property *being certain*; or the doxastic property *being believed*.

Thus, some sentences are well-formed, others are not; some sentences (propositions) are true, others are not; some sentences (propositions) are contingent, others are not; some objects are beautiful, others are not; some sentences (propositions) are evident, others are not; some sentences (propositions) are believed, others are not.

This means we should start by stating Chisholm's two questions in general terms:

1a. Which objects in general have property of interest *F* and which do not?

- Which sentences are well-formed, and which are not?
- Which sentences (propositions) are true, and which are not?
- Which sentences (propositions) are contingent, and which are not?
- Which objects are beautiful, and which are not?
- Which sentences (propositions) are evident, and which are not?
- Which sentences (propositions) are believed, and which are not?

2a. How do we decide if <u>this</u> object has property of interest *F* or not?

- How do we decide if <u>this</u> sentence is well-formed, or not?
- How do we decide if <u>this</u> sentence (proposition) is true, or not?

[9] Chapter 29 of Cusmariu 2026 contains a detailed analysis of the private language argument.
[10] See Albritton 1966 and Hertzberg 2022.
[11] Ch. 21 of my 2026 defends the view that there are aesthetic properties. I felt obligated to defend this view as a Platonist who makes art (sculpture). The book responds to Plato's objections to art in Ch. 9.

- How do we decide if <u>this</u> sentence (proposition) is contingent, or not?
- How do we decide if <u>this</u> object is beautiful, or not?
- How do we decide if <u>this</u> sentence (proposition) is evident, or not?
- How do we decide if <u>this</u> sentence (proposition) is believed, or not?

10. A Famous Diagnosis

In his Introduction to Wittgenstein's *Tractatus*, Russell applied the methodology of "On Denoting" to summarize the book:

> Starting from the principles of Symbolism and the relations which are necessary between words and things in any language, [*Tractatus*] applies the results of this inquiry to various departments of traditional philosophy, showing in each case how traditional philosophy and traditional solutions arise out of ignorance of the principles of Symbolism and out of misuse of language.

Here is Wittgenstein (in translation) with pretty much the same diagnosis:

> The book deals with the problems of philosophy, and shows, I believe, that the reason why these problems are posed is that the logic of our language is misunderstood.

11. "Hold on, there"

ME
What's on your mind, Socrates?
SOCRATES
"The logic of our language?" We spoke to the issue in our *Sophist*.
ME
Yes, I remember, language as "the interweaving of Forms."
SOCRATES
Was Wittgenstein aware of our view?
ME
By his own admission, Wittgenstein was indifferent to the history of philosophy.[12]
SOCRATES
Pity.

[12] In the "Author's Preface" of *Tractatus*, Wittgenstein stated (in translation) that "… the reason why I give no sources is that it is a matter of indifference to me whether the thoughts that I have had have been anticipated by someone else."

12. Continuing...

The problem of the criterion does not "arise out of ignorance of the principles of Symbolism." On the contrary, it has counterparts in mathematical logic, whose practitioners never grasped the connection, including Russell himself, who was doubtless familiar with these counterparts by the time he reviewed the *Tractatus* – the last volume of *Principia Mathematica* was published in 1913. Why, despite his vast knowledge of the history of philosophy, Russell seemed unaware of the problem of the criterion is a mystery. Wittgenstein, by his own admission, was indifferent to the history of philosophy.

Quine, who was a logician as well as a philosopher, also seemed unaware of the problem of the criterion, at least in Quine 1969.[13] Gödel 1944 did not see that his most famous result solved the problem of the criterion. Russell's theory of descriptions (Russell 1904) seems to have been a "paradigm" (Ramsey 1965, 263) only for philosophy, even though it started life as a definition of the uniqueness quantifier, $\exists!xFx =\text{df} (\exists x)(Fx \,\&\, (y)Fy \rightarrow (y{=}x))$ – which Russell did not mention in "On Denoting."

13. Déjà Vu

Criteria C1 and C2 have counterparts in mathematical logic.

- The characterization problem makes precise Criterion C1, which Chisholm termed "particularist."
- The decision problem makes precise Criterion C2, which Chisholm termed "methodist."

Here is mathematical logician Joseph R. Schoenfield on these problems. However, like Russell, Wittgenstein, Quine and Gödel, he also didn't "put two and two together":

> The primary object of a formal system is to provide a framework for proving theorems. Hence a particularly important problem for any formal system *F* is: find a necessary and sufficient condition that a formula of *F* be a theorem of *F*. This is called the *characterization problem* for *F*. There is a trivial solution to the characterization problem for a theory *T*: a formula is a theorem iff it has a proof. This is an unsatisfactory solution because the condition for **A** to be a theorem

[13] The opening lines of Quine 1969 read: "Epistemology is concerned with the foundations of science. Conceived thus broadly, epistemology includes the study of the foundations of mathematics." I agree with Quine to the extent that he thinks epistemology should include as one of its main tasks the clarification of scientific and mathematical knowledge. See Cusmariu 2012 and 2016.

> depends upon upon all formulas which might appear in a proof of **A**. In a satisfactory solution the condition must depend only upon **A** and formulas closely related to **A**.[14]

> A *decision method* for a formal system *F* is a method by which, given a formula of *F*, we can decide in a finite number of steps whether or not it is a theorem of *F*. The *decision problem* for *F* is the following: find a decision method for *F* or prove that no such method exists. … a decision method must be mechanical… a mechanical method is one which could be carried out by a suitably designed machine.[15]

Criterion C1 calls for a characterization property in Schoenfield's sense.

> Criterion C1: A property *G* of theory *T* in which *F* is a property of interest such that, if object *a* of *T* has *F*, then necessarily, *a* has *F* if and only if *a* has *G*.[16]

14. An Important Ambiguity

I could just as easily have stated Criterion C1 as Schoenfield would have, as a material biconditional:

> Criterion C1a: A property *G* of theory *T* in which *F* is a property of interest such that, if object *a* of *T* has *F*, then *a* has *F* if and only if *a* has *G*.

Another possibility is to state Criterion C1 as a material biconditional with characterization property *G* as an essential property:

> Criterion C1b: A property *G* of theory *T* in which *F* is a property of interest such that, if object *a* of *T* has *F*, then *a* has *F* if and only if necessarily, *a* has *G*.

Kripke semantics[17] made modality mathematically uncontroversial, so C1 as well as C1b will work in mathematical logic. A merely contingent criterion, C1a, would be inappropriate in mathematics.

C1b entails C1 on the assumption that if *a* has *F* essentially, then the proposition that *a* has *F* is necessarily true – though not the other way around. So, one key question is whether Criterion C1 is sufficient for present purposes. I will assume as much unless refuted.

Another key question is which particularist criterion version Chisholm would have needed to state the problem of the criterion. This is unclear from his

---

[14] Schoenfield 1967, 41, original italics.
[15] Schoenfield 1967, 106, original italics. There is no mention of Turing.
[16] By assumption, *F* and *G* are distinct properties. What it means for properties to be distinct is an interesting and difficult philosophical problem. It is discussed in Chapter 27 of my 2026.
[17] Kripke's earliest contribution to modal logic is Krikpe 1959.

formulation of the problem. For example, does he expect answers to his two questions

1. *What* do we know?
2. How are we to decide, in any particular case, *whether* we know?

to be necessarily true or just contingently true? Contingently true answers would be as inappropriate in philosophy as in mathematics.

15. Continuing...

Schoenfield's property of interest *F* in formal systems is *being a theorem*, whose objects are formulas. Here is his characterization property *G*:

JRS. A formula **A** of a theory *T* is a theorem of *T* if and only if it is valid in *T*.[18]

- Schoenfield seemed unaware that his informal explication of "valid" (p. 18) as "all the meanings of **A** are true in *T*" raises philosophical issues of concern to Quine.
- Schoenfield's comment (p, 133) raises a philosophical issue of concern in epistemology: "An interesting question is whether every mathematical truth… can be proved from axioms which are evidently true. However, we cannot hope to make progress with this question until we understand more clearly what is meant by being evidently true."
- It is a good question what difference it would have made had Schoenfield seen the connection between the above two problems of mathematical logic and the problem of the criterion.[19]

16. An Interlude

SOCRATES
Before moving on, it should be made clear that what you are calling a characterization property does not do away with the fact that it is a property – what we call a Form.
ME
Yes, every characterization property is a "One in Many."
SOCRATES

[18] Schoenfield 1967, 43. Informally, formula **A** is valid in *T* just in case it is true in *T* under every interpretation.
[19] Philosophers appear to be unaware that the problem of the criterion impacts the epistemology of mathematics. See Steiner 1975 and Kitcher 1984.

It should also be made clear that the characterization property *being valid* only explains the predication content of "is a theorem," not predication-as-such, which is still in need of an *aitia*. Our view is that the Form *being a theorem* provides such an *aitia*.

ME

Point well taken as well. Unfortunately, mathematics and logic use the notation "*Fx*," disregarding the implicit use of the "is" of predication. They claim that it is not necessary to take note of predication to get on with the business at hand.

SOCRATES

That amounts to pretending that philosophical problems are insignificant, does it not?

ME

Yes, it does. There's a lot of that going around, not just in mathematics.

SOCRATES

What are you doing about it?

ME

I wrote a book titled *Echoes from Plato's Cave* that documents the impact of philosophy, including your Theory of Forms, on science, mathematics, culture, society, and even art (at least my own).

SOCRATES

Good luck with your book.

17. Casting a Wider Net[20]

Examples of characterization properties elsewhere in philosophy, in mathematics, science, logic and even religion will show that the scope of the problem of the criterion is far greater than anything the literature on the problem seems to have realized.

- A proposition has the property *being true* if and only if it has characterization property *G* of a theory of truth, such as *correspondence to reality*. For Descartes, the criterion of truth, *G*, is *being perceived clearly and distinctly* (the C&D Rule). This criterion also applies to *being evident* and *being certain*.
- An action has the property *being morally permissible* if and only if it has characterization property *G* of a moral theory, such as *being universalizable* (Kant) or *maximizing utility* (Mill).

[20] I realize that the problem of the criterion also impacts my semantic epistemology. For me, *being a theorem* is the characterization property of semantically evident mathematical propositions. The issues are complex and require separate treatment.

- A geometric figure has the property *being a circle* if and only if it has characterization property G of a definition, such as *having all points equidistant from the center*.
- A number has the property *being prime* if and only if it has characterization property G of a definition, such as *being a natural number greater than 1 that is not a product of two smaller natural numbers*.
- A scientific generalization has the property *being lawlike* if and only if it has characterization property G, such as *being capable of confirmation* or *supporting counterfactuals*.
- An event has the property *being a miracle* if and only if it has characterization property G, such as *being attributed to causes believed to be supernatural*.
- A formula has the property *being well-formed* if and only if it has a (complex) syntactic characterization property, G, defined in a formal system.

18. A Particularist Criterion

Criterion C1 calls for a characterization property.[21] Here are the details once again.

> Criterion C1: A property G of theory T in which F is a property of interest such that, if object a of T has F, then, necessarily, a has F if and only if a has G.

19. A Methodist Criterion

Can the answer to 1a,

> 1a. Which objects in general have property of interest F and which do not?

also answer 2a,

> 2a. How do we decide if this specific object has property F or not?

Well, why can't G also be the property that will help us decide if and only if this object has property F or not? After all, G is satisfied in the general case; therefore, it must be satisfied by any specific case as well, including this one?

That is true but beside the point. The question is not whether this object satisfies property G or not. It does. The question is how we can decide whether this object satisfies property G and by inference property F.

We have arrived at the conclusion that a different kind of criterion is needed. Thus, the phrase "the problem of the criterion" is a misnomer because it suggests that there is only one criterion. Properly understood, then, what we have here is "the problem of criteria."

[21] Thus, I disagree with the literature that criteria are necessarily epistemic principles. For example, characterization properties are not necessarily epistemic.

- Question 1a calls for a criterion in the form of a property, in this case, a characterization property *G* necessarily associated with objects that have property of interest *F.*
- Question 2a calls for a criterion in the form of a method for deciding if the characterization property *G* is satisfied by this object or not; so that, if it is satisfied, it can be inferred that this object has *F* based on the relationship between *F* and *G* already spelled out.

Criterion C2 calls for a method. Here are the details:

Criterion C2: A method *M* of theory *T* in which *F* is a property of interest of objects of *T* such that, necessarily, *M* is necessary and sufficient for deciding whether or not a specific object of *T* has characterization property *G* of *T.*

Criterion C2 is a method in Schoenfield's sense. It calls for a finite sequence of steps sufficient to yield the answer to a "yes/no" question if carried out correctly to completion. More formally, *M* can be thought of as a Turing machine (De Mol 2025 and Petzold 2008).

20. So What?

A requirement in analytic philosophy since Plato is that philosophical problems should be presented in argument form[22] to determine whether a proposition of interest is true or whether it is false.[23] So, if the problem of criteria is to be more than an intriguing puzzle, what argument-form version would address the "so what?" requirement?[24]

Here are our two criteria again.

Criterion 1: A property *G* of theory *T* in which *F* is a property of interest such that, if object *a* of *T* has *F*, then, necessarily, *a* has *F* if and only if *a* has *G.*

Criterion 2: A method *M* of theory *T* in which *F* is a property of interest of objects

---

[22] I did not find a statement of the problem of the criterion in argument form in any of the following: Sosa 2007; Fumerton 2011; Aikin 2010 and 2011; Williams 2001; and Moser 1983.

[23] The problem of the criterion is not discussed in Bruce & Barbone 2011, which analyzes 100 philosophical arguments.

[24] Russell's Paradox proved that this requirement also applies to philosophical problems stated in counterexample form, such as the Gettier Problem, which showed that traditional definition of knowledge as justified true belief in inadequate. Instead of argumentation, there followed a veritable stampede of "repairs" to the traditional definition of knowledge. Unlike most of the literature, Cusmariu 2023 presented the Gettier Problem in argument form. De Poe 2011 uses some of the same assumptions I did but, unlike my paper, does not show that the Gettier Problem works by deriving a contradiction. He also does not suggest which assumptions can be questioned. I became aware of this article only recently.

of *T* such that, necessarily, *M* is necessary and sufficient for deciding whether or not a specific object of *T* has characterization property *G* of *T*.

Criteria 1 and 2 are stated in general terms, so it is advisable to state the problem of the criterion also in general terms to avoid the objection that key premises are only true in a specific field, such as epistemology, as well as the objection that a potential fallacy of ambiguity is lurking. Logical possibility needs to be made explicit for the sake of clarity.

A. It is logically possible for Criterion C2 to be applied to decide whether or not a specific proposition of theory *T* has property *F* of interest in theory *T* only if it is logically possible that Criterion C1 has already been applied to distinguish in general propositions of theory *T* that have property *F* of interest in theory *T* from propositions of theory *T* that do not have *F*.

B. It is logically possible for Criterion C1 to be applied to distinguish in general propositions of theory *T* that have property *F* of interest in theory *T* from propositions of theory *T* that do not have *F* only if it is logically possible that Criterion C2 has already been applied to decide whether or not a specific proposition of theory *T* has property *F* of interest in theory *T*.

We can derive objectives O1 and O2 from A and B:

O1. To determine by applying Criterion C1 *what* we know in the general case, i.e., to determine by applying Criterion C1 which propositions of theory *T* in general have property *F* of interest in theory *T* and which propositions of theory *T* do not have *F*.

O2. To decide by applying Criterion C2 *whether* we know in a specific case, i.e., to decide by applying Criterion C2 whether or not a specific proposition of theory *T* has property *F* of interest in theory *T*.

Here is a statement of the problem of the criterion in argument form:

1. If A and B are both true, then, criteria C1 and C2 presuppose each other's applicability.

2. If criteria C1 and C2 presuppose each other's applicability, then, owing to self-evidently vicious circularity, skepticism is warranted that we can achieve objective O1 and skepticism is warranted that we can achieve objective O2.

Therefore,

3. If A and B are both true, then, skepticism is warranted that we can achieve objective O1 and skepticism is warranted that we can achieve objective O2.

4. A and B are both true.

Therefore,

5. Skepticism is warranted that we can achieve objective O1 and skepticism is warranted that we can achieve objective O2.

21. A Way Out?

As the saying goes, "one man's *modus ponens* is another man's *modus tollens*." First, we deny Step 5 and assert its negation,

6. Either skepticism is unwarranted that we can achieve objective O1 or skepticism is unwarranted that we can achieve objective O2.

The next move is to assert,

7. Skepticism is warranted that we can achieve objective O2,

and infer from 6 and 7,

8. Skepticism is unwarranted that we can achieve objective O1.

from which it follows that A and B are not both true. If 7 is true, then B is false.

For this strategy to work, we must be able to justify Step 7. That is, we must be able to justify the proposition that skepticism is warranted that we can achieve objective O2,

O2. To decide by applying Criterion C2 *whether* we know in a specific case, i.e., to decide by applying Criterion C2 whether or not a specific proposition of theory *T* has property *F* of interest in theory *T*.

22. Gödel's Proof[25]

Schoenfield's characterization property is easily stated in the sentential calculus (SC).

SC1. A formula **A** of SC is a theorem of SC if and only if **A** is valid in SC – i.e., true in SC under every interpretation.

What about a version of Criterion C2 in SC?

SC2. A method *M* of *SC* in which the property *F*, *being a theorem*, is a property of interest such that, necessarily, *M* is necessary a sufficient for deciding whether or not characterization property *G* of SC is exemplified by specific sentence *s* of SC.

This is also easy: The truth table method is necessary and sufficient for deciding whether or not a formula **A** is valid in SC and, accordingly, is a theorem of SC.

[25] An informal exposition is Nagel, Newman and Hofstadter 2001. This book saw no connection between Gödel's result and to the problem of the criterion. Gödel 1931 is the original paper.

Though SC is consistent, complete and decidable, it is not adequate for the purpose of deriving elementary arithmetic, let alone the whole of mathematics. For that, a more powerful formal system is necessary, Zermelo-Fraenkel set theory with the axiom of choice, ZFC.

The characterization property for ZFC is the same as for SC, so that Criterion C1 is easily met:

> ZFC1. A formula **A** of ZFC is a theorem of ZFC if and only if **A** is valid in ZFC – i.e., true in ZFC under every interpretation.

What about a methodist criterion in ZFC?

> ZFC2. A method *M* of ZFC in which the property *F*, *being a theorem of ZFC*, is a property of interest such that, necessarily, *M* is necessary and sufficient for deciding whether or not a specific sentence of ZFC exemplifies the characterization property *G* of ZFC, *being valid in ZFC*.

Gödel's First Incompleteness Theorem shows that there are – indeed, there must be – undecidable sentences in formal systems such as ZFC. That is, there are sentences of ZFC that are valid in ZFC but are such that neither they nor their negations are demonstrably syntactic consequences in ZFC.[26] It follows that there are sentences of ZFC that are such that it is not necessarily true that there is a decision method for proving either that they or their negation have the characterization property *being valid in ZFC* and by implication have the property *being a theorem of ZFC*.

Among the many philosophical implications of Gödel's startling result is that an assumption without which the problem of the criterion cannot even be stated is false:

> (I) There must be a methodist criterion for every particularist criterion.[27]

Accordingly, Gödel's Proof can rescue from the clutches of skepticism only particularist criteria.[28]

A project for another time is to study the following options:

> Option 1: Work with a Criterion C2 that drops the modal requirement.

> Option 2: Work with a link between Criterion C2 and Criterion C1 that is weaker than (I).

---

[26] If we write "semantic consequence" instead, Gödel's Proof collapses into the Liar Paradox.

[27] The converse is unaffected by Gödel's Proof.

[28] Specifying a characterization property can be challenging. For example, Logical Positivism failed because *being verifiable* was easily shown not to be a characterization property of *being meaningful*.

Option 3: Adopt a theory dependent, case-by-case approach to Criterion C2.

23. Wither Chisholm's Epistemology?

The property of greatest interest in Chisholm's epistemology, which is a property of propositions for him, is *being known by person S*. So, under Criterion 1, what is the characterization property *G* such that, necessarily, a proposition *h* has the property *F*, *being known by person S* if and only if *h* has *G*?

Perhaps Chisholm can take as the characterization property *G, satisfying the conditions of D2,* which is the definition of knowledge in Chisholm 1987, 98:

> D2 h is known by S = Df (1) h is true; (2) S accepts h; (3) h is evident for S; and (4) if h is defectively evident for S, then h is implied by a conjunction of propositions each of which is evident for S but not defectively evident for S.

This choice of *G* won't do because Criterion C1 calls for a property *G* such that, necessarily, a proposition has the property *being known by S* if and only if it has *G*. However, it is not necessarily the case that a proposition has the property *being known by S*, *F*, if and only if it has the property *G*, *satisfying the conditions of D2*.

Why not?

It does not follow that a proposition does not have the property *F*, *being known by S*, because it does not satisfy the conditions of D2. After all, D2 is one of many possible definitions of knowledge, as well as one of many possible solutions to the Gettier Problem. All that follows if a proposition does not satisfy the conditions of D2 is that it does not have the property *being D2-known by S.* This property is not the same property as the property *F*, *being known by S*.

We should ask: If the property *satisfying the conditions of D2* is not a characterization property of the property of greatest interest in Chisholm's epistemology, *being known by S*, what is the point of D2?

Let us move on to see how he would fare with Criterion C2, which in this case calls for specifying a method *M* such that, necessarily, *M* is necessary and sufficient for deciding whether or not a specific knowledge claim has the "right" characterization property. What method M would enable Chisholm to decide whether or not a specific knowledge claim has, for example, the characterization property*, satisfying condition (4) of D2*,

> (4) if *h* is defectively evident for S, then *h* is implied by a conjunction of propositions each of which is evident for *S* but not defectively evident for *S*?

However, Chisholm would not necessarily have considered these outcomes a serious problem for his epistemology. His view in Chisholm 1988 is that (a) the problem of the criterion is not solvable without begging the question, and that (b) it is a problem with which every epistemology is stuck. Tellingly, the opening comment of Chisholm 1988 (231) echoes his Aquinas Lecture: "I am convinced that, so far as the problem of the criterion is concerned, most philosophers – and most epistemologists – prefer to bury their heads in the sand."[29] I would add, "most mathematical logicians as well."[30]

24. Hegel Weighs In

Scott F. Aikin (2010, 379) listed eight conditionals to express what he takes to be the implications of the problem of the criterion for Hegel's epistemology.

> 1. If we are to solve the problem of the criterion, the criterion must come from within cognition.
>
> 2. If a criterion is from cognition, it must be in terms of cognition's historically situated satisfactions.
>
> 3. Cognition is satisfied only if knowledge is complete.
>
> 4. Knowledge is complete only if it is systematic.
>
> 5. Knowledge is systematic only if it is made explicit by philosophy.
>
> 6. If a system is made explicit by philosophy, both the system and the articulation must be circular and ongoing.
>
> 7. If philosophy's articulations are circular and ongoing, then they are procedurally infinite.
>
> 8. If philosophy is to be procedurally infinite, it must be practiced in a cultural-political climate of an open society with a state protecting freedoms.

Several points:

- Combining the antecedent of first conditional with the consequent of the last conditional yields: "If we are to solve the problem of the criterion, it [philosophy] must be practiced in a cultural-political climate of an open society with a state protecting freedoms." But, what will protect us if the state fails to protect freedoms? Yes, it's a rhetorical question. Marxists love Hegel.
- As noted above, the particularist criterion is ontological, while the methodist criterion is epistemological. Thus, the first step of Aikin's sequence of conditionals is false because its implicitly epistemological consequent, "the

[29] If Chisholm is right, we should find that few if any philosophers down through the ages have formulated particularist and methodist criteria for key properties of interest in their theories.
[30] It would take us too far afield to spell out the implications of Criterion C2 for the fields of inquiry noted earlier in connection with Criterion C1.

criterion must come from within cognition," is false. While methodist criteria are implicitly epistemological, they cannot be "procedurally infinite" because proofs are finite. Particularist criteria are not "procedurally infinite" either.

- From step (7) we can conclude that Hegel would have considered circularity unavoidable in philosophy and therefore would have rejected the assumption that circularity is "self-evidently vicious." As far as analytic philosophy is concerned, this check-mates Hegel.

25. The Cartesian Circle[31]

Echoing Chisholm, Van Cleve (1979) has stated that Descartes' Cartesian Circle "in its generalized form, is none other than the Problem of the Criterion, a problem that any epistemology must face."

Let us put this claim to a test. We start with Van Cleve's statement of the Circle in his 1979, 55-6:

> The problem of the Cartesian Circle arose for Descartes because he appeared to commit himself to each of the following propositions:
>
> (1) I can know (be certain) that (*p*) whatever I perceive clearly and distinctly is true only if I first know (am certain) that (*q*) God exists and is not a deceiver.
>
> (2) I can know (be certain) that (*q*) God exists and is not a deceiver only if I first know (am certain) that (*p*) whatever I perceive clearly and distinctly is true.
>
> Obviously, if (1) and (2) are both true, I can never be certain of either *p* or *q*. To be certain of either I would already have to be certain of the other. Yet Descartes said he was certain of *both p* and *q*. How can this be possible? Any adequate solution to the problem of the Cartesian Circle will plainly have to deny either (1) or (2).

26. So What?

Let us capture this material in argument form, which the Cartesian Circle literature has not done.[32]

We start with p and q, which are as Van Cleve indicated:

(a) Person *S* can know (be certain) that *p* <u>only if</u> *S* first knows (is certain) that *q.*

- This is Van Cleve's (1) in generalized form.
- *p* and *q* are as above, suitably rephrased.

(b) Person *S* can know (be certain) that *q* <u>only if</u> *S* first knows (is certain) that *p*.

[31] Descartes "evil genius" takes up Ch. 28 of my 2026.

[32] I did not find a statement of the Cartesian Circle in argument form in any of two dozen sources I consulted.

- This is Van Cleve's (2) in generalized form.
- *p* and *q* are as above, suitably rephrased.

Therefore – from (a) and (b),

(c) If it is the case that (a) and (b) are both true, then person S can never be certain of either p or q.

- The inference from (a) and (b) to (c) is implied by Van Cleve's "obviously" comment.
- However, it is not at all obvious that (c) is either a syntactic or a semantic consequence of (a) and (b).[33]
- Syllogistic logic cannot derive (c) from (a) and (b) because there is no categorical proposition in standard form – an A, E, I or U proposition – equivalent to (c).
- The argument can proceed with (c) as an "obvious" assumption.

Therefore – from (c), by contraposition,

(d) If person *S* can sometimes be certain of either *p* or *q*, then it is not the case that (a) and (b) are both true.

(e) Descartes asserted that he was certain of both *p* and *q*.

- Van Cleve stated as much.
- If Descartes was certain of both *p* and *q*, then, presumably he was certain <u>sometimes</u> of both *p* and *q*.

Therefore – from (d) and (e)

(f) It is not the case that (a) and (b) are both true.

(g) Descartes asserted that (a) and (b) are both true.

- This step comes from Van Cleve's statement that Descartes "appeared to commit himself to each of the following propositions," meaning (1) and (2), i.e., (a) and (b).

Therefore – from (f) and (g), by conjunction

(h) It is not the case that (a) and (b) are both true <u>and</u> Descartes asserted that (a) and (b) are both true.

[33] I studied Descartes and Kant with Jim Van Cleve at Brown University in the 1970s. He was one of my PhD thesis advisers along with Ernie Sosa and Chisholm. I raised the problematic inference of (c) from (a) and (b) with Jim in conversation at the time. He shrugged and repeated the "obvious" comment, as I recall.

Therefore – from (h)

(i) Descartes was inattentive, sloppy, confused, wrong.

or

(j) Descartes anticipated Moore's Paradox.

27. Inattentive…

- Alternative conclusion (i) is as much an embarrassment for Descartes as for his commentators because its derivation follows from the application of an elementary rule of logic.
- Are we to conclude that the extensive literature on the Cartesian Circle – clever, elaborate, imaginative – was for the sake of rescuing Descartes' reputation?

28. Moore's Paradox

Alternative conclusion (j) makes Descartes look much better. It places the Cartesian Circle in the context of an interesting problem raised by G.E. Moore and named after him.[34]

Abbreviations and a diagram will come in handy:

- ˜BP for "I don't believe that P."
- ˜B˜P for "I don't believe that not-P."
- B˜P for "I believe that not-P."
- BP for "I believe that P.

P & ˜BP	P & B˜P
<u>M1a: Omissive</u>	<u>M2a: Comissive</u>
˜P & ˜B˜P	˜P & BP
<u>M1b: Omissive</u>	<u>M2b: Comissive</u>

- M2b is the logical form of step (h) of the argument.
- The literature, including Moore and Williams, appears not to have recognized M1b and M2b as Moore-sentence types, missing the connection to the Cartesian Circle.

29. Two Arguments

[34] See Williams 2023. This book was published posthumously. Williams died in 2019.

Van Cleve did not explain what he had in mind by the "generalized form" of the Cartesian Circle; nor which version of the problem of the criterion he claimed the "generalized form" of the Cartesian Circle instantiated.

Two arguments can be generated to put this claim to a test. Argument 1 mirrors my paraphrase of Van Cleve's presentation of the Cartesian Circle; Argument 2 mirrors my own argument for skepticism.

Argument 1

We start with versions of *p* and *q*. Here are Van Cleve's originals:

- *p:* "Whatever I perceive clearly and distinctly is true."
- *q*: "God exists and is not a deceiver."

As a generalized version of Van Cleve's original *p*, *w*e can use "Whatever person *S* perceives clearly and distinctly is true."

What about a generalized version of *q*? Descartes had something specific in mind using "God exists and is not a deceiver" in the Circle, such as calling upon God to guarantee the accuracy of our memories or to guarantee the metaphysical certainty of clear and distinct perceptions. It's unclear what else could do that. Perhaps a Turing machine? A powerful movie computer such as HAL 9000 (*2001: A Space Odyssey*) or Colossus (*The Forbin Project*)? Argument 1 can't even get started.

Argument 2

To get Argument 2 going, we need first generalized versions of Van Cleve's original *p* and *q*.

- We can use "Whatever person *S* perceives clearly and distinctly is true" as a generalized version of Van Cleve's *p*.
- To mirror my own argument for skepticism, let us use by way of replacement of Van Cleve's *q* the following*:* "What I <u>now</u> perceive clearly and distinctly is true."

Next, here are our two criteria for ease of reference:

<u>Criterion 1</u>: A property *G* of theory *T* in which *F* is a property of interest such that, if object *a* of *T* has *F*, then, necessarily, *a* has *F* if and only if *a* has *G*.

<u>Criterion 2</u>: A method *M* of theory *T* in which *F* is a property of interest of objects of *T* such that, necessarily, *M* is necessary and sufficient for deciding whether or not a specific object of *T* has characterization property *G* of *T*.

Recall that the problem of the criterion starts out with two claims:

A. It is logically possible for Criterion C2 to be applied to decide whether or not a specific proposition of theory *T* has property *F* of interest in theory *T* <u>only if</u> it is logically possible that Criterion C1 has already been applied to distinguish in general propositions of theory *T* that have property *F* of interest in theory *T* from propositions of theory *T* that do not have property *F* of interest in theory *T*.

B. It is logically possible for Criterion C1 to be applied to distinguish in general propositions of theory *T* that have property *F* of interest in theory *T* from propositions of theory *T* that do not have property *F* of interest in theory *T* <u>only if</u> it is logically possible that Criterion C2 has already been applied to decide whether or not a specific proposition of theory *T* has property *F* of interest in theory *T*.

Instantiating A and B yields (1') and (2'), respectively, and Argument 2 can proceed.

(1') It is logically possible for me to decide by applying Criterion C2 that (*q*) what I <u>now</u> perceive clearly and distinctly is true <u>only if</u> it is logically possible for me to have determined already in the general case by applying Criterion C1 that (*p*) whatever I perceive clearly and distinctly is true.

(2') It is logically possible for me determine in the general case by applying Criterion C1 that (*p*) whatever I perceive clearly and distinctly is true <u>only if</u> it is logically possible for me to have already decided by applying Criterion C2 that (*q*) what I <u>now</u> perceive clearly and distinctly is true.

The objectives here are analogous:

O3. To determine by applying Criterion C1 to the general case that (*p*) whatever I perceive clearly and distinctly is true.

O4. To decide by applying Criterion C2 to a specific case that (*q*) what I <u>now</u> perceive clearly and distinctly is true.

Therefore,

3. If (1') and (2') are both true, then, criteria C1 and C2 presuppose each other's applicability.

4. If criteria C1 and C2 presuppose each other's applicability, then, owing to self-evidently vicious circularity, skepticism is warranted that we can achieve objective O3 <u>and</u> that we can achieve objective 4.

Therefore,

5. If (1') and (2') are both true, then, skepticism is warranted that we can achieve objective O3 <u>and</u> skepticism is warranted that we can achieve objective O4.

6. (1') and (2') are both true.

Therefore,

> 7. Skepticism is warranted that we can achieve objective O3 and skepticism is warranted that we can achieve objective O4.

A *modus tollens* counter argument can escape the Circle by denying 7 and asserting its negation 8,

> 8. Either skepticism is unwarranted that we can achieve objective O3 or skepticism is unwarranted that we can achieve objective O4.

Gödel's incompleteness result proved that methodist skepticism is warranted. The upshot it that Descartes cannot escape the Circle and secure methodist as well particularist criteria, which would have been his preferred outcome. Gödel can meet a fellow mathematician only half way.

30. Outstanding Questions

- *Being a theorem* is a property of a theory T of greater interest than *being well-formed* in part because of the impact their characterization properties have on the objectives of T. How do we rank-order properties of interest in a theory, any theory?
- Criteria C1 and C2 for being such that position and momentum are simultaneously computable can be applied in classical mechanics provided characterization properties can be formulated. In quantum mechanics, however, only C1 can be applied because the uncertainly principle $\sigma X \, \sigma P \geq \hbar/2$ implies that standard deviations for position σX and momentum σP can in principle be made arbitrarily small for them taken singly but not both simultaneously. What are the implications of this unavoidable difference for the prospects of a unified field theory?
- Hilbert's 1902 statement of the 24th problem alludes to Criterion C1 as well as Criterion C2: "Criteria of simplicity, or proof of the greatest simplicity of certain proofs. Develop a theory of the method of proof in mathematics in general." So, how many of his 24 problems call for C1 and how many for C2?
- Finally, what is the impact of limitations on C2 owing to Gödel's Proof?[35]

References

Aikin, Scott. 2011. "The Regress Argument for Skepticism." In *Just the Arguments: 100 of the Most Important Arguments in Western Philosophy,* edited by Michael Bruce and Steven Barbone, 146-151. Malden, MA: Wiley-Blackwell.

[35] Many thanks to Kevin McCain for helpful comments on previous drafts.

-----------. 2010. "The Problem of the Criterion and Hegel's Model for Epistemic Infinitism," *History of Philosophy Quarterly* 27 (4): 379-388.

Albritton, Rogers. 1966. "On Wittgenstein's Use of the Term 'Criterion'." In *Wittgenstein: The Philosophical Investigations*, edited by George Pitcher, 231-250. New York: Anchor Books.

Amico, Robert P. 1993. *The Problem of the Criterion.* Lanham MD: Rowman and Littlefield.

Bruce, Michael and Steven Barbone (eds.). 2011. *Just the Arguments: 100 of the Most Important Arguments in Western Philosophy.* Malden, MA: Wiley-Blackwell.

Chisholm, Roderick M. 1973. *The Problem of the Criterion*. Milwaukee: Marquette University Press.

------------. 1957. *Perceiving: A Philosophical Study.* Ithaca: Cornell University Press.

------------. 1989. *Theory of Knowledge*, third edition. Englewood Cliffs, New Jersey: Prentice Hall

------------. 1988. "Reply to Amico on the Problem of the Criterion." *Philosophical Papers* XVII, 3: 231-234.

------------. 1973 & Swartz, Robert J. (eds.) *Empirical Knowledge: Readings from Contemporary Sources.* Englewood Cliffs, NJ: Prentice Hall.

Cusmariu, Arnold. 2012. "Toward a Semantic Approach in Epistemology," *Logos & Episteme. An International Journal of Epistemology* III, 4: 531-543.

------------. 2016. "Semantic Epistemology Redux: Proof and Validity in Quantum Mechanics. *Logos & Episteme. An International Journal of Epistemology* VII, 2: 287-304.

------------. 2023. "Is JTB Knowledge Hopeless?", *Logos & Episteme* XIV. *An International Journal of Epistemology*, 3: 261-270.

------------. 2026, forthcoming. *Echoes from Plato's Cave*. Singapore: Jenny Stanford Publishing.

De Mol, Liesbeth. 2025. "Turing Machines." *The Stanford Encyclopedia of Philosophy* https://plato.stanford.edu/entries/turing-machine/. Accessed 7-15-25.

De Poe, John M. 2011. "Gettier's Arguments against the Traditional Account of Knowledge." In *Just the Arguments: 100 of the Most Important Arguments in Western Philosophy*, edited by Michael Bruce and Steven Barbone, 156-158. Malden, MA: Wiley-Blackwell.

Empiricus, Sextus. 1935. *Against the Logicians*, trans. R.G. Bury (Loeb edition). (London: W. Heinema.

Feldman, Richard and Feldman, Fred. 2023. “Roderick Chisholm.” *The Stanford Encyclopedia of Philosophy.* https://plato.stanford.edu/entries/chisholm/ Accessed 7-25-25.

Fumerton, Richard. 2011. “The Problem of the Criterion.” In *The Oxford Handbook of Skepticism,* edited by John Greco, 34-52. Oxford: The University Press.

Gödel, Kurt. 1944. “Russell’s Mathematical Logic.” In *The Philosophy of Bertrand Russell,* edited by Schilpp, P.A., 123-154. LaSalle IL: Open Court.

------------. 1931 (1967). “Über formal unentscheidbare Sätze der Principia Mathematica und verwandter Systeme I,” *Monatshefte für Mathematik Physik*, 38: 173–198. English translation in *From Frege to Gödel: A Source Book in Mathematical Logic, 1879–1931,* edited by Van Heijenoort, J., 596-616. Cambridge, MA: Harvard University Press.

Hahn, Lewis E. (ed). 1997. *The Philosophy of Roderick M. Chisholm.* Chicago: Open Court.

Hertzberg, Lars. 2022. *Wittgenstein on Criteria and Practices.* Cambridge: Cambridge University Press.

Hilbert, David. 1902. “Mathematical Problems,” *Bulletin of the American Mathematical Society*, 8(10): 437–479.

Kitcher, Philip. 1984. *The Nature of Mathematical Knowledge*, Oxford: Oxford University Press.

Kripke, Saul. 1959. “A completeness theorem in modal logic.” *Journal of Symbolic Logic* 24.1: 1–14.

Lehrer, Keith (ed.). 1975. *Analysis and Metaphysics: Essays in Honor of R.M. Chisholm*. New York: Springer.

McCain, Kevin. No date. “The Problem of the Criterion.” Internet Encyclopedia of Philosophy. https://iep.utm.edu/problem-of-the-criterion/ Accessed 8-10-25.

Moser, Paul K. 1983. *Philosophy After Objectivity*. New York: Oxford University Press.

Nelson, Leonard. 1965. *Socratic Method and Critical Philosophy*. New York: Dover Publications.

Nagel, Ernest, Newman, James R. and Hofstadter, Douglas R. 2001. *Gödel’s Proof.* New York: New York University Press.

Petzold, Charles. 2008. *The Annotated Turing*. Indianapolis: Wiley Publishing.

Quine, W.V.O. 1969. “Epistemology Naturalized,” originally published in his *Ontological Relativity and Other Essays*, New York: Columbia University Press.

Ramsey, Frank. 1965. *The Foundations of Mathematics*. Totowa, New Jersey: Littlefield, Adams & Company.

Russell, Bertrand. 1945. *The History of Western Philosophy.* New York: Simon and Schuster.
------------. 1905. "On Denoting". *Mind* 14 (56): 479–493.
Schoenfield, Joseph R. 1967. *Mathematical Logic*. Menlo Park, CA: Addison-Wesley.
Sosa, Ernest (ed.). 1979. *Essays on the Philosophy of Roderick M. Chisholm*. Amsterdam: Rodopi.
------------. 2007. *A Virtue Epistemology: Apt Belief and Reflective Knowledge*, Volume 1. Oxford: Clarendon Press (2007).
Steiner, Mark. 1975. *Mathematical Knowledge.* Ithaca: Cornell University Press.
Van Cleve, James. 2011. "Sosa on easy knowledge and the problem of the criterion," *Philosophical Studies* 153: 19-28.
------------. 1979. "Foundationalism, Epistemic Principles, and the Cartesian Circle." *The Philosophical Review* 88: 55-91.
Wittgenstein, Ludwig. 1961 (1921). *Tractatus Logics-Philosophicus.* Translated by D.F. Pears & B.F. McGuinness. London: Routledge & Kegan Paul.
Williams, John N. 2023. *A Unified Treatment of Moore's Paradox: Belief, Knowledge, Assertion and Rationality*. Oxford: Oxford University Press.
Williams, Michael. 2001. "The Problem of the Criterion." In his *Problems of Knowledge*, 155-157. Oxford: The University Press.