A NON-AXIOMATIC SYSTEM CAN DEAL WITH APPARENT NONMONOTONICITY IN THE SAME WAY AS HUMAN BEINGS

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ABSTRACT: Lukowski argued that four typical examples of inferences used to show that human beings' natural reasoning is nonmonotonic do not reveal that. Lukowski's analyses support the idea that those inferences are actually monotonic deductions. My aim here is to check whether a particular non-axiomatic logic is consistent with the habitual conclusions people draw in those kinds of inferences. This is relevant because that non-axiomatic logic is the logical structure of a computer program. So, if the logic is coherent with the usual conclusions in those types of inferences, the computer program is also compatible with them.

KEYWORDS: classical logic, inheritance relation, non-axiomatic system, nonmonotonicity, term logic

Introduction

When we try to relate logic to human thought, we promptly find a problem: nonmonotonicity. In classical logic, given a set of premises coming to a conclusion, if one more premise is added to that set, that should not modify the conclusion. In other words, the derivation of the initial conclusion should keep being possible. Formally, this can be expressed as follows.

Let 'A' be a set of well-formed formulae in propositional calculus. Let 'α' be a well-formed formula in propositional calculus not included in 'A' (i.e., $\alpha \notin A$). Let 'β' be one more well-formed formula in propositional calculus.

(1) If $\{A\} \vdash \beta$ then $\{A, \alpha\} \vdash \beta$

Where '$\vdash$' represents logical deduction.

However, daily human reasoning does not always respect (1). An example (coming from Lukowski 2013) can show it.

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Let 'r_1', 'r_2', 'r_3', 'r_4', and 'r_5' the results in, respectively, five medical tests. Let 'I_1' and 'I_2' be two illnesses. After getting 'r_1', 'r_2', 'r_3', and 'r_4', physicians can accept (2).

(2) $\{r_1, r_2, r_3, r_4\} \vdash I_1$

Nevertheless, one more test can give 'r_5', which can lead physicians to reject 'I_1' because it is more linked to I_2. Hence, physicians accept (3) and (4) too.

(3) $\{r_1, r_2, r_3, r_4, r_5\} \nvdash I_1$

Where '$\nvdash$' expresses that there is no logical deduction.

(4) $\{r_1, r_2, r_3, r_4, r_5\} \vdash I_2$

This appears to be a clear example of nonmonotonicity. But Lukowski (2013) presents arguments supporting the idea that the nonmonotonicity (2), (3), and (4) reflect is apparent. Classical logic can explain what underlies facts such as these ones.

Lukowski (2013) describes three more types of apparent nonmonotonic inferences. Lukowski also proposes that classical logic can capture them. My aims here are not to challenge those accounts, explain them, or develop them. I will only try to argue in favor of the hypothesis that a computer program can also capture the conclusions people usually derive in the four types of inference Lukowski (2013) presents. That computer program works in a similar way as our mind. It is NARS (Non-Axiomatic Reasoning System; e.g., Wang 2006).

The first section will explain what the four kinds of apparent nonmonotonic inferences Lukowski (2013) addresses are. The second one will consider the components of NARS required to deal with those four types of inferences. The last section will show that NARS can get the same conclusions as people in those very kinds of inferences.

Four types of apparent nonmonotonic inferences

The first type is that indicated above, that is, that related to (2), (3), and (4). Resorting to the material conditional in classical logic, (5) can represent it as well.

(5) $[(r_1 \wedge r_2 \wedge r_3 \wedge r_4) \Rightarrow I_1] \wedge \neg[(r_1 \wedge r_2 \wedge r_3 \wedge r_4 \wedge r_5) \Rightarrow I_1] \wedge [(r_1 \wedge r_2 \wedge r_3 \wedge r_4 \wedge r_5) \Rightarrow I_2]$

In (5), '$\wedge$' stands for conjunction and '$\neg$' represents negation. The material conditional is expressed as '$\Rightarrow$', and not, as usual, as '$\rightarrow$', because '$\rightarrow$' is a symbol in NAL (Non-Axiomatic Logic; Wang 2013). NAL is the logical non-axiomatic system constituting the architecture of NARS. Thus, I will use '$\rightarrow$' below in a sense different from that of the conditional in propositional calculus.

The second kind of apparent nonmonotonic inference refers to a situation in which two friends are going to meet with each other in a pub. One of them, for example, 'Friend$_1$', must complete several actions to come to the pub: Friend$_1$ has to go out, take a taxi, etc.

Let 'a_1', 'a_2', 'a_3', and 'a_4' be the actions Friend$_1$ must complete before coming to the pub. Let 'p' be the act to arrive to the pub. This leads to (6).

(6) $\{a_1, a_2, a_3, a_4\} \vdash p$

Suppose that Friend$_1$ receives bad news: for example, the friend that was going to be waiting in the pub had an accident. This causes Friend$_1$ not to go to the pub.

Let 'n' be the bad news Friend$_1$ receives. This situation can be expressed as in (7).

(7) $\{a_1, a_2, a_3, a_4, n\} \nvdash p$

This is another case of apparent nonmonotonicity. For this case, a formula can be built as well: (8).

(8) $[(a_1 \wedge a_2 \wedge a_3 \wedge a_4) \Rightarrow p] \wedge \neg[(a_1 \wedge a_2 \wedge a_3 \wedge a_4 \wedge n) \Rightarrow p]$

Although the structure of (8) does not seem to be very different from that of (5), the solution Lukowski (2013) offers from classical logic for these two types of inferences is not the same. Something similar is what happens with the third kind of inference. Lukowski's (2013) example is that of a bird, for example, 'Bird$_1$'. If we know that Bird$_1$ is a bird, we can conclude that Bird$_1$ can fly. If, after that, we learn that Bird$_1$ is an ostrich, we have to rectify as ostriches cannot fly.

Let 'b' be the fact that Bird$_1$ is a bird. Let 'f' be the fact that Bird$_1$ can fly. The deduction is (9).

(9) $\{b\} \vdash f$

Let 'o' be the fact that Bird$_1$ is an ostrich. Then, we have (10).

(10) $\{b, o\} \nvdash f$

If we take (9) and (10) together, they also represent a case of apparent nonmonotonicity. It can be expressed by means of formula (11).

(11) $(b \Rightarrow f) \wedge \neg[(b \wedge o) \Rightarrow f]$

While the solution within classical logic Lukowski (2013) gives for the last kind of inference is, again, different, the structure of the fourth type does not appear to be away from those of the previous ones either. A person, for instance, 'Person$_1$' has a car. The car is parked in front of Person$_1$'s house. That causes one to think that

$Person_1$ is home. Later, it can be learned that $Person_1$ is not home (e.g., $Person_1$ was walking to a neighbor's house). Hence, the conclusion requires to be changed.

Let 'c' be the fact that $Person_1$'s car is in front of $Person_1$'s house. Let 'h' be the fact that $Person_1$ is home. The first inference is (12).

(12) $\{c\} \vdash h$

The apparent nonmonotonicity can be seen here by means of (13), which is the inference corresponding to the second stage (the stage in which it is known that $Person_1$ is not home).

(13) $\{c, \neg h\} \nvdash h$

The formula for (12) and (13) can be (14).

(14) $(c \Rightarrow h) \wedge \neg[(c \wedge \neg h) \Rightarrow h]$

As indicated, Lukowski (2013) appears to argue that formulae such as (5), (8), (11), and (14) do not represent real cases of nonmonotonicity in classical logic. But I will not address Lukowski's arguments in the present paper. I will check whether a computer program trying to make inferences in a similar way as us can also explain the conclusions of these four kinds of inferences. The program is NARS. Some of its components are described in the next section.

NARS, NAL: Some Characteristics

NARS is not the only computer program deriving conclusions in a way akin to that of human reasoning. For example, there are programs written in accordance with the theory of mental models (e.g., Johnson-Laird 2023; Johnson-Laird, Quelhas, and Rasga 2021). Accounts of some of the codes following the latter theory are to be found in papers such as Johnson-Laird, Byrne, and Khemlani (2023), Khemlani, Byrne, and Johnson-Laird (2018), or Khemlani and Johnson-Laird (2022).

The computer program I will consider is NARS, whose logic is NAL. For the aims of this paper, it is not required to take all the rules or the complete grammar of NAL into account. Some of its characteristics are enough. I will describe those characteristics below following their presentation in Wang (2013) (in this section, I will present NAL formulae literally or almost literally taken from Wang 2013; when this is the case, I will refer to the particular definition and page number in Wang 2013, in which the formula appears; I will not use quotation marks because that could create confusion with the symbols). NAL has different complexity levels: from NAL-1 to NAL-9. The higher the complexity of the level is, more rules the system has and more developed its grammar is. Here, a level such as NAL-2 suffices.

The chief characteristic of NAL is that it is a term logic. The relations linking its terms are inheritance relations. Typically, inheritance relations in NAL are expressed as in (15).

(15) $S \rightarrow P$ (f, c) (Wang 2013, 40, Definition 3.8)

The account of '$S \rightarrow P$' reveals that NAL is a term logic. The equivalences are the following (see Wang 2013, 14, Definition 2.2):

S $=_{df}$ subject

P $=_{df}$ predicate

$\rightarrow$ $=_{df}$ inheritance copula

The 'inheritance copula' refers to a relation between 'S' and 'P' that is 'reflexive and transitive'.

Let us suppose inheritance relations (16) and (17).

(16) Frog → Amphibian
(17) Amphibian → Animal

Inheritance relation (16) expresses that frog is a kind of amphibian. Inheritance relation (17) indicates that amphibian is a type of animal. Given that inheritance relations are transitive, in NAL, we can accept (18).

(18) {(16), (17)} ⊢ (19)
(19) Frog → Animal

Examples (16), (17), and (19) show how important some traditional concepts in logic and philosophy of science are in NAL. That is the case of, for example, 'extension' and 'intension' (see, e.g., Carnap 1947). The meanings of 'extension' and 'intension' within NAL are clear. In (16), 'frog' (the subject) is an element in the extension of 'amphibian' (the predicate); 'amphibian' (the predicate) is an element in the intension of 'frog' (the subject). In the same way, in (17), 'amphibian' (the subject) is an element in the extension of 'animal' (the predicate); 'animal' (the predicate) is an element in the intension of 'amphibian' (the subject). Likewise, in (19), 'frog' (the subject) is an element in the extension of 'animal' (the predicate); 'animal' (the predicate) is an element in the intension of 'frog' (the subject). So, 'extension' and 'intension' are defined by virtue of 'subject' and 'predicate' (see, e.g., Wang 2013, Definition 2.8).

As far as '(f, c)' in (15) is concerned, it has to do with the fact that NAL is a non-axiomatic logic. NAL is a logic of the latter kind because its main assumption is 'AIKR' ('Assumption of Insufficient Knowledge and Resources'). AIKR causes no inheritance relation to be absolutely true in the system (see also, e.g., Wang 2011).

There can be evidence supporting the relation (W^+) and evidence refuting the relation (W^-). The formulae corresponding to the types of evidence in NAL are in (20).

(20) $W^+ = (S^E \cap P^E) + (P^I \cap S^I)$, $W^- = (S^E - P^E) + (P^I - S^I)$, $W = W^+ + W^- = S^E + P^I$ (Wang 2013, 28, Definition 3.2)

In (20), superscripts 'E' and 'I' designate, respectively, 'extension' and 'intension'. So, for example, 'S^E' stands for the extension of term 'S', that is, the subject.

On this basis, '(f, c)' can be accounted for. 'f' and 'c' refer, respectively, to 'the frequency and the confidence' of the relation they follow between brackets. Formulae (21) and (22) allow determining their values.

(21) $f = W^+/W$ (Wang 2013, 29, Definition 3.3)
(22) $c = W/(W + k)$ (Wang 2013, 29, Definition 3.3)

Regarding 'k' in (22), it is a NAL constant. A natural way to deal with it is to assume '$k = 1$' (Wang 2013, 30).

So, inheritance relations (16), (17), and (19) are incomplete. They require values for 'f' and 'c'. To give them their values, we need to think about a fictional scenario in which the system has considered certain number of frogs and amphibians.

Let us suppose that the system has considered five frogs and three toads. Let us suppose that the system has noted that both the five frogs and the three toads are amphibians. Let us suppose that the system has noted that the eight amphibians are animals. (16) and (17) should be expressed, respectively, as (23) and (24).

(23) Frog $\rightarrow$ Amphibian (1, 0.86)
(24) Amphibian $\rightarrow$ Animal (1, 0.89)

Transitivity comes from (16) and (17) to (19). In NAL-1, there are already formulae to calculate 'f' and 'c' for the conclusion of an inference taking formulae such as (23) and (24) as its premises. That is a deduction. Its formula is (25).

(25) $\{M \rightarrow P\ (f_1, c_1), S \rightarrow M\ (f_2, c_2)\} \vdash S \rightarrow P\ [(f_1 \times f_2), (f_1 \times c_1 \times f_2 \times c_2)]$

(This formula appears in Wang 2013, 52; I have modified 'f' and 'c' in the conclusion including information indicated in Wang 2013, 61, Table 4.7)

Given (25), from (23) and (24), it is possible to deduce (26).

(26) Frog $\rightarrow$ Animal (1, 0.77)

In NAL, inheritance relations can be from the subject to the predicate and from the predicate to the subject at once. The formula corresponding to those cases is '$S \leftrightarrow P$'. (27) offers a definition of the latter formula in meta-language.

(27) $(S \leftrightarrow P) \Leftrightarrow [(S \rightarrow P) \wedge (P \rightarrow S)]$ (Wang 2013, 77, Definition 6.1)

Where '$\Leftrightarrow$' (the biconditional symbol) and '$\wedge$' work as in classical propositional calculus.

The formula to calculate 'W^+' in the case of '$S \leftrightarrow P$' is not different from that of '$S \rightarrow P$'. However, the formula for 'W^-' is (28).

(28) $W^- = (S^E - P^E) + (P^E - S^E) + (P^I - S^I) + (S^I - P^I)$ (Wang 2013, 79, Definition 6.2)

Lastly, one more component of NAL important for my aims here is that the system also has a 'instance copula' (see, e.g., Wang 2013, 84, Definition 6.4). A way to express a subject whose extension has just one member is '{S}'. Thus, if the name of a particular frog is 'Kermit', that can be expressed as in (29).

(29) $\{Kermit\} \rightarrow P$

In (29), 'P' is an element of the intension of 'Kermit'. But the extension of 'Kermit' has only one element: the particular frog named 'Kermit'.

These few characteristics of NAL are enough to capture the usual conclusions in the apparent nonmonotonic inferences Lukowski (2013) presents. I will try to show this in the next section.

Four Kinds of Apparent Nonmonotonic Inferences from NAL

(5) describes the first type of inference Lukowski analyzes. To argue that NAL can account for people's habitual conclusion in it, it is necessary to express the content of (5) by means of the machinery of NAL. The first conjunct in (5) reveals that results 'r_1' to 'r_4' enable to think that the illness is 'I_1'.

Let '$R^{1\text{ to }4}$' be the concept referring to the cases in which the results obtained were 'r_1', 'r_2', 'r_3', and 'r_4'. Let 'f^+' and 'c^+' be high values for, respectively, 'f' and 'c' allowing accepting the statement they correspond to. The first conjunct in (5) can be expressed as in (30).

(30) $R^{1\text{ to }4} \rightarrow I_1\ (f^+, c^+)$

The second conjunct in (5) provides that, after knowing result 'r_5', physicians' opinion changes. Now, they think that 'I_1' cannot be the illness.

Let '$R^{1\text{ to }5}$' be the concept referring to the cases in which the results obtained were 'r_1', 'r_2', 'r_3', 'r_4', and 'r_5'. Let 'f^-' be a low value for 'f' leading to reject the statement it corresponds to. (31) can be the inheritance relation for the second conjunct.

(31) $R^{1\text{ to }5} \rightarrow I_1\ (f^-, c^+)$

The third conjunct in (5) indicates that ‘$R^{1\text{ to }5}$’ enables to assume that the illness is ‘I_2’. This can be claimed resorting to (32).

(32) $R^{1\text{ to }5} \rightarrow I_2\ (f^+, c^+)$

Or, if preferred, in an even stronger way, (33).

(33) $R^{1\text{ to }5} \leftrightarrow I_2\ (f^+, c^+)$

If (30), (31), and (32) or (33) are part of the knowledge of the system, NAL comes to the regular conclusion people derive in this kind of inference (for a case study describing the real way NAL works and changes its conclusions in medical contexts, see also, e.g., Wang and Awan 2011).

The well-formed formula in propositional calculus I have assigned to the second type of inference is (8). Its first conjunct states that there are several actions, ‘a_1’ to ‘a_4’, which Friend_1 has to complete to be able to go to the pub.

Let ‘$A^{1\text{ to }4}$’ be the term representing the set of those actions. The first conjunct of (8) could correspond to (34) in NAL.

(34) $A^{1\text{ to }4} \rightarrow \text{Pub}\ (f^+, c^+)$

What the second conjunct establishes is that bad news, ‘n’, can prevent Friend_1 from going to the pub.

Let ‘$A^{1\text{ to }4,\ n}$’ be the term whose extension includes ‘a_1’ to ‘a_4’ plus ‘n’. In NAL, the second conjunct can be (35).

(35) $A^{1\text{ to }4,\ n} \rightarrow \text{Pub}\ (f^-, c^+)$

If the system knows (34), (35), and that both ‘a_1’ to ‘a_4’ and ‘n’ are the case, it has to conclude that Friend_1 will not meet with the other friend in the pub.

In the example of the third type of inference, a particular bird, Bird_1, must be considered. Hence, (36) must hold in NAL.

(36) $\{\text{Bird}_1\} \rightarrow \text{Bird}$

The formula in classical logic attributed to this kind of inference is (11). Its first conjunct points out that birds can fly. That is also what (37) means.

(37) $\text{Bird} \rightarrow \text{Fly}\ (f^+, c^+)$

The second conjunct reveals that the system needs further information. It requires to know that, while Bird_1 is a bird, Bird_1 is an ostrich, and ostriches cannot fly. Thus, the system has to add (38), (39), and (40).

(38) $\{\text{Bird}_1\} \rightarrow \text{Ostrich}$
(39) $\text{Ostrich} \rightarrow \text{Bird}\ (f^+, c^+)$
(40) $\text{Ostrich} \rightarrow \text{Fly}\ (f^-, c^+)$

If the information in (37) to (40) is obtained in the order indicated, that is, if (38) to (40) are learned after knowing (37), NAL should conclude that $Bird_1$ cannot fly. This is because (40) updates the system, and 'f' is significantly lower in (40) than in (37).

The example for the last kind of inference presents a scenario with a person, $Person_1$, $Person_1$'s car, and $Person_1$'s house. $Person_1$'s car is in front of $Person_1$'s house. From this datum, it is inferred that $Person_1$ is home.

Let '$Car^{Person1}$' be the term referring to the set of circumstances in which $Person_1$'s car is in front of $Person_1$'s house. Let '$Home^{Person1}$' be the term referring to the set of circumstances in which $Person_1$ is home. The initial belief can be (41).

(41) $Car^{Person1} \rightarrow Home^{Person1}$ (f^+, c^+)

Nonetheless, the information that $Person_1$ is not actually home can change the value of 'f' in (41). It can lead to an inheritance relation such as (42).

(42) $Car^{Person1} \rightarrow Home^{Person1}$ (f^-, c^+)

The values of 'f' in NAL are modified as the information is acquired. Therefore, in NAL it is possible first to accept (41) and then, when the information is updated, to reject (41) and accept (42).

Accordingly, NAL can explain people's common conclusions in apparent nonmonotonic inferences such as those Lukowski (2013) indicates. This means that NARS is able to make those inferences in the same way as human beings.

Conclusions

Lukowski (2013) presents four types of inferences that are often used as examples of nonmonotonicity. This is important. One might think that, if classical logic is monotonic and people's reasoning is nonmonotonic, classical logic cannot show how human reasoning is.

Lukowski's (2013) arguments seem to reveal that those kinds of inferences are not nonmonotonic. Lukowski's idea appears to be that, if human reasoning moves away from logic because of its nonmonotonicity, that is not the case by virtue of those four examples of inferences. The examples seem to keep within monotonicity and classical logic.

I have analyzed those examples with a different goal. I wanted to check whether NAL can explain the habitual conclusions to which people come with those types of inferences. The relevance of this is that NAL is the logic NARS follows. So, if NAL captures the derivation processes related to the inferences, we can say that a computer program such as NARS can simulate human thought, at least with regard to the four kinds of inferences Lukowski (2013) addresses.

My arguments appear to show that NAL can deal with the four types of inferences reviewed. The key is the link between NAL and AIKR. New information can change what is acceptable. New knowledge can modify a value such as frequency, that is, ‘f’. Frequency in NAL refers to inheritance relations built by means of the concepts of extension and intension. But the extension and intension of terms may not be the same in different times. They depend on our knowledge at any time. Furthermore, the literature about both NAL and NARS is extensive. So, it is very likely that accounts akin to those I have given above are to be found in that literature, albeit expressed differently and referring to other aspects or topics.

In any case, both NAL and the human mind change what they consider to be true as they learn. Hence, one might expect that their behavior is similar with regard to the four kinds of inferences Lukowski’s (2013) describes. This paper has tried to show that is the case.

References

Carnap, R. 1947. *Meaning and Necessity: A Study in Semantics and Modal Logic*. Chicago: The University of Chicago Press.

Johnson-Laird, P. N. 2023. “Possibilities and human reasoning.” *Possibilities Studies & Society* 1(1-2): 105-112. https://doi.org/10.1177/27538699231152731

Johnson-Laird, P. N., Byrne, R. M. J., and Khemlani, S. 2023. “Human verifications: Computable with truth tables outside logic.” *Proceedings of the National Academy of Sciences* 120(40): e2310488120. https://doi.org/10.1073/pnas.2310488120

Johnson-Laird, P. N., Quelhas, A. C., and Rasga, C. 2021. “The mental model theory of free choice permissions and paradoxical disjunctive inferences.” *Journal of Cognitive Psychology* 33(8): 951-973. https://doi.org/10.1080/20445911.2021.1967963

Khemlani, S., Byrne, R. M. J., and Johnson-Laird, P. N. 2018. “Facts and possibilities: A model-based theory of sentential reasoning.” *Cognitive Science* 42(6): 1887-1924. https://doi.org/10.1111/cogs.12634

Khemlani, S., and Johnson-Laird, P. N. 2022. “Reasoning about properties: A computational theory.” *Psychological Review* 129(2): 289-312. https://doi.org/10.1037/rev0000240

Lukowski, P. 2013. “Is human reasoning really nonmonotonic?” *Logic and Logical Philosophy* 22: 63-73. https://doi.org/10.12775/LLP.2013.004

Wang, P. 2006. *Rigid Flexibility: The Logic of Intelligence*. Dordrecht: Springer. https://doi.org/10.1007/1-4020-5045-3

Wang, P. 2011. "The assumptions on knowledge and resources of rationality." *International Journal of Machine Consciousness* 3(1): 193-218. https://doi.org/10.1142/S1793843011000686

Wang, P. 2013. *Non-Axiomatic Logic: A Model of Intelligent Reasoning*. Singapore: World Scientific. https://doi.org/10.1142/8665

Wang, P., and Awan, S.. 2011. "Reasoning in Non-Axiomatic Logic: A case study in medical diagnosis." in *Artificial General Intelligence. AGI 2011. Lecture Notes in Computer Science, volume 6830*, edited by J. Schmidhuber, K. R. Thórisson, and M. Looks, 297-302. Heidelberg: Springer, Berlin, Heidelberg. https://doi.org/10.1007/978-3-642-22887-2_33