ABSTRACT: An argument structure that covers both cases Gettier described in his 1963 paper reinforces the conclusion of my 2012 Logos & Episteme article that the justified true belief (JTB) conception of knowledge is inconsistent. The stronger argument makes possible identification of fundamental flaws in the standard approach of adding a fourth condition to JTB, so that a new kind of skepticism becomes inevitable unless conceptual change occurs.

KEYWORDS: Gettier Problem, inconsistency, JTB knowledge, ostensible evidence, skepticism

Background

In articles I published in Logos & Episteme in 2012 and 2016, I presented and defended a semantic approach in epistemology intended to capture arguably our most valuable repositories of knowledge, mathematics and science. The 2012 article also stated one of two components of the Gettier Problem (GP) in argument form and explained how semantic epistemology dealt with GP. I am returning to the issue to formulate a stronger argument that covers both GP components, which reinforces the earlier conclusion that the justified true belief (JTB) conception of knowledge is inconsistent and reveals a fundamental flaw in the standard approach to GP of adding a fourth condition to JTB. A new kind of skepticism becomes inevitable unless conceptual change occurs.

A Precedent Missed

The literature on the Gettier Problem (GP) has been growing steadily since the 1963 publication of Edmund L. Gettier’s paper. Books, articles and collections of article have analyzed the problem and proposed solutions. A lengthy and detailed survey by Stephen Hetherington in the Internet Encyclopedia of Philosophy1 assessed contributions and remaining challenges.

Missing from the GP literature is awareness of a remarkably similar event some sixty years earlier that had even more devastating consequences. I refer to Bertrand Russell’s proof that Rule V in Frege’s Grundgesetze der Arithmetik was

1 https://iep.utm.edu/gettier/#H14/.

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inconsistent. In a 1902 letter to Frege (van Heijenoort 1967, 124-5), Russell showed that substituting the open sentence “~(x ε x)” in Rule V entails contradiction. Two important lessons were learned from Russell’s Paradox, both missed by the GP literature. First, counterexamples are inconsistency proofs and as such should be spelled out in argument form. Second, conceptual change may prove necessary. I will address both issues.

Gettier Text Analysis

Gettier presents two counterexamples to the analysis of knowledge as justified true belief (JTB). It is not explained why two counterexamples are necessary—why not three?—nor the difference between them, the implication being that two counterexamples are needed so that different objections can be raised. This is false as we shall see.

Gettier states at the outset that he will “argue”—as opposed to merely state, contend, hypothesize or claim—that a certain proposition is false, namely, that justified true belief is “sufficient” (1963, 121, original italics) for knowledge. As he notes, this proposition is one conjunct of JTB, according to which justified true belief is necessary and sufficient for knowledge. Gettier uses the term “argument” in the very next sentence, implying intent to defend the proposition that justified true belief is insufficient for knowledge by means of standard logic. The paper states at the end (1963, 123) that “the two examples show” that justified true belief is not sufficient for knowledge, implying by the use of “show” that an argument has been presented for this conclusion.

So, do we find in Gettier’s paper an argument in the standard logical sense whose conclusion is that JTB is insufficient for knowledge? We do not.

2 Inconsistent sentences can be of the form “p & ~p” or of the form “p ≡ ~p”. Russell proved the latter and went on to solve the problem he had discovered. Gettier never “returned to the scene of the crime.”
3 The the naive conception of a set was eventually replaced with the iterative conception of a set (see Boolos 1971). I attempted to rescue naive sets using a “patch” approach in Cusmariu 1979.
4 On a personal note, I recall an episode in a Chisholm seminar at Brown University in the mid-70s during which he recalled his first reaction to GP, frustration evident in his tone: “Well, all you have to do is say this …” Chisholm never stated GP in argument form, in class or in his books; nor did he seem to realize that a much more devastating conclusion had been proved. Like everyone else at the time, he focused on repairing the analysis.
5 Gettier does not use the term “counterexample”; he uses the term “Cases.” The earliest use of the term “counterexample” seems to be in Clark 1963.
What we have instead are what are called “points,” which Gettier says he is “noting.” Here is the language (1963, 121):

I shall begin by noting two points. First, in the sense of “justified” in which S’s being justified in believing P is a necessary condition of S’s knowing that P, it is possible for a person to be justified in believing a proposition that is in fact false. Secondly, for any proposition P, if S is justified in believing P, and P entails Q, and S deduces Q from P and accepts Q as a result of this deduction, then S is justified in believing Q.

This passage is a muddle. For example, what are we to make of the locution “in the sense of ‘justified’ in which …” What sense is that? The text does not say. To get a handle on what is being asserted, let us list component propositions:

(a) If S knows P, then S is justified in believing P.
(b) If S is justified in believing P, and P entails Q, and S deduces Q from P and accepts Q as a result of this deduction, then S is justified in believing Q.
(c) It is it is possible for a person to be justified in believing a proposition that is false.

The passage explicitly links (c) with (a), leaving (c)-(b) links up in the air. I can think of three ways (c) and (a) could be linked, all of which seem to me false:

1. Propositions (a) is meaningful only if proposition (c) is true.
2. Propositions (a) is true only if proposition (c) is true.
3. Propositions (a) entails proposition (c).

If not 1-3, how are (a) and (c) to be linked? The text does not say.

What about (c) and (b)? Should (b) likewise be read this way:

(b*) If S is justified in believing P, and P entails Q, and S deduces Q from P and accepts Q as a result of this deduction, then S is justified in believing Q “in the sense of ‘justified’ in which it is possible for a person to be justified in believing a proposition that is false”?

The text offers no comment. Interpretations 1-3 are also false if applied to (b).

I propose ignore these complications and interpret the first “point” of the passage as stating as an assumption that it is possible for a proposition to be justified and false, omitting potential links to (a) and (b), which are unnecessary for the purpose of building an argument. As to what “possible” means, we can make do with “we can imagine circumstances in which …” the circumstances being Gettier’s two Cases.
A Two-Stage Argument

The same argument structure is sufficient to cover both Cases.

Stage I: Proving that Smith knows that Q.

1. The proposition that P is evident for Smith.

   Case I: P is the proposition that Jones will get the job, and Jones has ten coins in his pocket.

   Comment: Gettier states (p. 122) that P is evident for Smith because “the president of the company assured him that Jones would in the end be selected” and because Smith “counted the coins in Jones’s pocket ten minutes ago.”

   Case II: P is the proposition that Jones owns a Ford.

   Comment: Gettier states (1963, 122) that P is evident for Smith because “Jones has at all times in the past within Smith’s memory owned a car, and always a Ford, and that Jones has just offered Smith a ride while driving a Ford.”

2. P logically implies Q.

   Case I: Q is the proposition that the man who will get the job has ten coins in his pocket.

   Comments: Gettier describes (1963, 122) the logical connection between P and Q as entailment, suggesting that P logically implies Q. This is not obvious.

   • Proposition P has the form $(\exists x)(\exists y)((x=a \land Fx) \land (y=b \land Gy) \land x=y)$ – where $a$ and $b$ are individual constants co-designating Smith and $F$ and $G$ are the predicates “will get the job” and “has ten coins in his pocket,” respectively.

   • Proposition Q has the form $(\exists ! x)(\exists ! y)(Fx \land Gy) \land (Fy \land Hy) \land x=y$ – where $\exists !$ is a uniqueness quantifier, $F$ and $G$ are the predicates “is a man” and “will get the job,” respectively; and $H$ is the predicate “has ten coins in his pocket.”

   • It seems clear that logical form by itself is not sufficient to show that P logically implies Q. However, let’s not worry about additional assumptions needed to get the entailment to go through and grant that the doxastic burden is not so onerous that Smith cannot recognize the entailment.

   Case II: Q is the proposition that Jones owns a Ford or Brown is in Barcelona.

   Comment 1: Gettier mentions two more versions Q, about Brown being in Boston or in Brest-Litovsk. However, Gettier focuses only on the Barcelona Q, so I propose to ignore the other versions.

   Comment 2: Q is follows logically from P as a substitution instance of an elementary rule of the propositional calculus, Disjunction.
3. Smith recognizes the inference from P to Q.
4. Smith accepts Q.

*Comment:* Gettier states that Smith accepts Q in both Cases on the basis of recognizing the inference from P to Q.

5. Q is true.
6. For any propositions X and Y, if X is evident for a person S, X logically implies Y, S recognizes the inferences from X to Y, and S accepts Y on the basis of recognizing this inference, then Y is evident for S.

*Comment:* Step 6 is Gettier second “point.”

Therefore, by instantiation and *Modus Ponens* from Steps 1, 2, 3, 4 and 6:
7. Q is evident for Smith.
8. The JTB conception of knowledge entails the following:
   - For any proposition X, if X is true, S accepts X, and X is evident for S, then S knows X.

Therefore, by instantiation and *Modus Ponens* from Steps 5, 4, 7 and 8:
9. Smith knows that Q.

**Stage II: Proving that Smith does not know that Q.**

We start by citing the language Gettier provides to support his claim that Smith does not know that Q in both Cases.

**Case I** (1963, 122):

But imagine, further, that unknown to Smith, he himself, not Jones, will get the job. And, also, unknown to Smith, he himself has ten coins in his pocket. Proposition (e) is then true, though proposition (d), from which Smith inferred (e), is false. In our example, then, all of the following are true: (i) (e) is true, (ii) Smith believes that (e) is true, and (iii) Smith is justified in believing that (e) is true. But it is equally clear that Smith does not know that (e) is true; for (e) is true in virtue of the number of coins in Smith’s pocket, while Smith does not know how many coins are in Smith’s pocket, and bases his belief in (e) on a count of the coins in Jones’s pocket, whom he falsely believes to be the man who will get the job.

Let us use our P and Q symbolism and link to Gettier’s.

- P is Gettier’s (d), the proposition that Jones will get the job, and Jones has ten coins in his pocket.

*Comment:* Recall that P has this logical form: \((\exists x)(\exists y)((x=a & Fx & (y=b & Gy) & x=y))\) – where a and b are individual constants co-designating Smith and F and G are the predicates “will get the job” and “has ten coins in his pocket.”
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respectively.

- Q is Gettier’s (e), the proposition that the man who will get the job has ten coins in his pocket.

Comment: Recall that Q has this logical form: $(\exists! x)(\exists! y)(Fx \& Gx \& (Fx \& Hy) \& x=y)$ – where $\exists!$ is the uniqueness quantifier, $F$ and $G$ are the predicates “is a man” and “will get the job,” respectively; and $H$ is the predicate “has ten coins in his pocket.”

Using our P and Q symbolism yields this paraphrase:

But it is equally clear that Smith does not know that Q is true; for Q is true in virtue of the number of coins in Smith's pocket, while Smith does not know how many coins are in Smith’s pocket, and bases his belief in Q on a count of the coins in Jones's pocket, whom he falsely believes to be the man who will get the job.

Comment: I will simplify this convoluted explanation shortly.

Case II (1963, 123):

But imagine now that two further conditions hold. First, Jones does not own a Ford, but is at present driving a rented car. And secondly, by the sheerest coincidence, and entirely unknown to Smith, the place mentioned in proposition (h) happens really to be the place where Brown is. If these two conditions hold then Smith does not know that (h) is true, even though (i) (h) is true, (ii) Smith does believe that (h) is true, and (iii) Smith is justified in believing that (h) is true.

Let us use our P and Q symbolism here as well and link to Gettier’s.

- P is Gettier’s (f), the proposition that Jones owns a Ford.
- Q is Gettier’s (h), the proposition that Jones owns a Ford or Brown is in Barcelona.

Using our P and Q symbolism yields this paraphrase:

But imagine now that two further conditions hold. First, P is false. And secondly, by the sheerest coincidence, and entirely unknown to Smith, the place mentioned in proposition Q happens really to be the place where Brown is. If these two conditions hold then Smith does not know that Q is true.

We are ready to start the argument at Stage II.

11. If Q is true by coincidence (accident, happenstance, luck) and Smith bases his belief that Q on an inference from a false but evident belief that P, then Smith does not know that Q.

Comment: Step 11 sums up the factors that, on my interpretation, drive Gettier’s claim that Smith does not know Q in both Cases.
12. Q is true by coincidence (accident, happenstance, luck) in both cases.

- **Case I:** Gettier has established that it is a coincidence that Smith happens to have ten coins in his pocket.
- **Case II:** Gettier has established that it is a coincidence that Brown happens to be in Barcelona.

13. Gettier has established that Smith based Q on an inference from P in both Cases.

14. P is false but evident in both Cases.

- **Case I:** Gettier has established that it is evident for Smith that Jones will get the job, adding that Jones will not get the job; so that in the circumstances described, it is false but evident that Jones will get the job.

- **Case II:** Similarly, Gettier has established that it is evident for Smith that Jones owns a Ford, adding that Jones no longer owns a Ford but is driving a rented car, meaning that in the circumstances described; so that it is false but evident that Jones owns a Ford.

Therefore, by instantiation and *Modus Ponens* from Steps 11-14:

15. Smith does not know that Q in either Case.

Therefore, by *Conjunction*:

16. In both Cases, Smith knows that Q and Smith does not know that Q.

17. The JTB conception of knowledge is inconsistent.

**Ways Out**

1. **So What?**

In epistemology and analytic philosophy generally, studying a concept means providing an analysis in terms of necessary and sufficient conditions—NS for short. The following considerations apply.

I. An NS is possible for any concept.

II. An NS is necessary for the purpose of applying a concept correctly.

III. A person must to be in possession of an NS to apply a concept correctly.

IV. Something of practical value is gained by having an NS of a concept.

V. Something of theoretical value is gained by having an NS of a concept.

I-IV are false or doubtful. Responding to GP by rejecting V, i.e., doing without an NS of propositional knowledge, seems extreme or at least premature.
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For one thing, it would not answer Plato’s question in the *Meno* about the
difference between knowledge and true belief.

2. The “Patch” Approach

The standard approach has been to accept that the JTB conception of knowledge
has been refuted and attempt repairs by adding a fourth condition.\(^6\) However, the
next Gettier counterexample may be just around the corner—the “wild goose
chase” worry—leading to increasingly complicated JTB+ conceptions of
knowledge, so complicated that they risk distorting pre-analytic intuitions to the
point that one cannot tell what knowledge means under various revisions or how
to test them.

To put matters another way, consider the contrapositive of 11:

11a. If Smith knows that Q, then either it is not the case that Q is true by
coincidence (accident, happenstance, luck) or it is not the case that Smith bases his
belief that Q on an inference from a false but evident belief that P.

A repair of JTB that implemented the consequents of Step 11—of which
there are quite a few in the literature (see Hetherington)—is not a substitute for a
proof of consistency. The GP literature is not even aware that such a proof is
necessary.

3. The Ostensibly Evident

Regarding Gettier’s first “point,” I’d like to suggest that we restrict “evident” to true
propositions and apply “ostensibly evident” to false propositions. This has the
following consequences:

- **Case I:** The proposition that Jones is the man who will get the job is only
  ostensibly evident for Smith if Smith will get the job rather than Jones.

- **Case II:** The proposition that Jones owns a Ford is only ostensibly evident for
  Smith if Jones does not own a Ford but is driving a rented car.

- An epistemic principle whose antecedent contains ostensibly evident
  propositions and can only transmit ostensible evidence. Thus, if P is false in
  both Cases, Step 6 (Gettier’s second “point”) must be restated to read “For any
  propositions X and Y, if X is ostensibly evident for a person S, X logically
  implies Y, S recognizes the inference from X to Y, and S accepts Y on the basis
  of recognizing this inference, then Y is ostensibly evident for S.”

\(^6\) That is the approach taken by Chisholm, one of Gettier’s targets, in the three editions of his
*Theory of Knowledge* (1966, 23; 1977, 105-110; and 1989, 91-100). Interestingly, A.J. Ayer, the
other philosopher Gettier targeted, seems to have shown no interest in answering GP; at least,
not in print.
Accordingly, Step 8 must be restated to read "For any proposition \( X \), if \( X \) is true, \( S \) accepts \( X \), and \( X \) is ostensibly evident for \( S \), then \( S \) ostensibly knows \( X \)."

So, GP is about a new category of ostensible knowledge, the other category being claims to know something that turns out to be false.

False evidence also raises this worry: While \( Q \) is true in both Gettier Cases, these propositions have been inferred from false but evident propositions so, in a sense, they are... vacuously true!

Look at it this way: We cannot be said to know what is false; so how is it that we can be said to know on the basis of false propositions?

Wider Implications

Some of the consequences of restricting "evident" to truths in a semantic conception of scientific and mathematical knowledge are addressed in Cusmariu 2012 and 2016. Exploring the implications of this restriction epistemology-wide would require book-length treatment. Two other issues are topics for another time:

- How to prove that the semantic conception of knowledge is consistent; and, if it is consistent, whether it is complete in mathematics in light of Gödel's discoveries.

- Whether epistemic properties are properties of propositions according to semantic epistemology or properties of persons according to virtue epistemology (see Baehr, Code 1987, Sosa 1991, Plantinga 1992, and Montmarquet 1993).

The concept of ostensible evidence deserves consideration independently of the Gettier Problem—just as iterative sets would have been a good idea independently of the Russell Paradox.\(^7\) Should it turn out, however, that epistemology is stuck with a “patch” approach, an argument for skepticism would result to the effect that the JTB knowledge has gaps of unknowable number—Gettier Cases—and therefore we JTB-know less than we think we know. This kind of skepticism is very different from the kind philosophers have considered since Descartes and might well prove to be the proverbial “straw that breaks the camel’s back” of the JTB conception of knowledge.\(^8\)

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\(^7\) I would argue that any theory proposed as a solution to a problem should satisfy this requirement, in science as well as philosophy.

\(^8\) David Christensen, Christopher Hill and Gary Rosenkrantz provided helpful comments on earlier drafts.
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References


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