A NOTE ON "A CONNEXIVE CONDITIONAL"

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ABSTRACT: In a recent article, Mario Günther presented a conditional that is claimed to be connexive. The aim of this short discussion note is to show that Günther's claim is not without problems.

KEYWORDS: connexive logic, conditionals, Aristotle's thesis, Boethius' thesis, strongly connexive logic, consequential implication, contra-classical logic, many-valued logic, constructive logic, negation inconsistency

As Mario Günther writes in (2022, 52), connexive logics are characterized by the following theses, together with the invalidity of $(A \to B) \to (B \to A)$.

Aristotle's theses: $\neg(\neg A \to A)$, $\neg(A \to \neg A)$

Boethius' theses: $(A \to B) \to \neg(A \to \neg B)$, $(A \to \neg B) \to \neg(A \to B)$

Given this definition of connexive logic, introduced by the modern founder of this topic, namely Storrs McCall in (1963; 1966), and followed by Wansing (2022), Günther's conditional is *not* connexive. It does, however, have some connexive flavour to it. Let us now turn to explain this in some details by pointing to some related developments.

What Günther follows at the beginning of (2022, §2) is a relatively recent suggestion made by Andreas Kapsner in (2012), requiring not only the above connexive theses, but also the following conditions.

UnSat1: In no model, $(A \to \neg A)$ is satisfiable, and neither is $(\neg A \to A)$, (for any A).

UnSat2: In no model, $(A \to B)$ and $(A \to \neg B)$ are satisfiable simultaneously (for any A and B).

The resulting systems that satisfy both of these additional conditions, together with the connexive principles, are called *strongly connexive logics*. Note also that logics that satisfy the Unsat principles are labeled as *Kapsner strong* by Luis Estrada-González and Elisángela Ramírez-Cámara in (2016, 347). With these notions in mind, Günther's system is Kapsner strong and satisfies Aristotle's thesis, but *not* Boethius' thesis.

We should note further that there are a number of systems in the literature that follow the same pattern of not being connexive, but being Kapsner strong and that satisfy Aristotle's thesis. First, and most importantly, systematic investigations by Claudio Pizzi, later partly with Timothy Williamson, must be acknowledged. In (1977), Pizzi develops the first system that can be seen as following the above pattern. Note also that Pizzi was the first, to the best of our knowledge, who introduced the following variant of Boethius' thesis, originally called *conditional Boethius' thesis* in (1977), and later called *weak Boethius' thesis* in (1996):[1]

Weak Boethius' thesis: $(A \rightarrow B) \supset \neg(A \rightarrow \neg B)$.

Then, since (1977), Pizzi developed a number of systems that are Kapsner strong and satisfy Aristotle's thesis as well as Weak Boethius' thesis, but *not* Boethius' thesis (the fact that the systems are Kapsner strong is not observed by Pizzi himself, but it can be easily confirmed by simple calculations). Moreover, Pizzi (1977, 289) discusses the conditional considered by Günther, and thus the conditional discussed in (Günther 2022) is not novel to Günther. Other examples that follow the same pattern include Graham Priest's system in (1999) as well as more recent investigations into variations of strict implication by Guido Gherardi and Eugenio Orlandelli in (2021; 2022). In footnote 6 of (Günther 2022), Priest's method is described as more complicated, but this is not the case. Priest's recipe is exactly the same as the one by Günther (2022) in which the antecedent of a conditional is required to be possible. Priest does also consider another version requiring in addition that the consequent of a conditional is not necessary, but that is not terribly more complicated either. Unfortunately, Günther (2022) does not define a notion of semantic consequence. Priest (1999) considers two such definitions for a language containing the conditional advocated by Günther. The familiar definition has the consequence of invalidating $p \rightarrow p$ for atomic formulas p, whereas building the satisfiablilty constraint into the definition of entailment results in a system that is neither monotonic nor closed under uniform substitution. The former property may be seen as casting doubt on the applicability of the promoted conditional in natural language semantics, whilst the failure of closure under uniform substitution casts doubt on the logicality of the system defined.

Before closing, here are three more remarks. First, what might be interesting to note, though not stressed by Günther, is that a simple variant of Lewis' conditional will bring us to the realm of *contra-classical logics* (cf. (Humberstone 2000)). The

[1] This was not acknowledged by Kapsner in (2012) in which connexive logics that do not satisfy the Unsat principles are labelled, perhaps unfortunately, as weakly connexive logics in contrast to strongly connexive logics.

same applies to the variants of strict implications explored by Gherardi and Orlandelli, and this seems to be a simple and interesting route to contra-classicality. Second, what remains to be an interesting challenge is to devise a strongly connexive logics that enjoys an intuitive semantics. Note here that strongly connexive logics exist, as noted by Kapsner (2012), since Angell and McCall's four-valued logic **CC1** is an example. But, it is far from enjoying an intuitive semantics. On the other hand, the system **C**, introduced by Wansing (2005), enjoys an intuitive semantics (cf. (Priest 2008, 178)), but it is not strongly connexive.[2] Therefore, the problem remains open to find a system that is strongly connexive and has an intuitive semantics.[3] Third, Günther's endorsement of the allegedly connexive conditional seems to be driven by his view that conditionals with contradictory antecedents are "not exactly intelligible" and that "non-trivial reasoning from inconsistent premises poses at least a challenge for intelligibility." However, this seems to be exactly the challenge posed by the presence of the system **C** since it enjoys an intuitive semantics, but is also negation inconsistent *without* being trivial. Moreover, Günther also holds (notation adjusted) that "the truth of $A \wedge \neg A \rightarrow B$ for any B is hardly intelligible," and seems to be welcoming that $A \wedge \neg A \rightarrow B$ is not valid with respect to his allegedly connexive conditional. But, how about the case with the entailment? If the familiar definition is taken, then one will still have $A \wedge \neg A \vDash B$. Is this intelligible? If not, Günther may prefer Priest's alternative suggestion, or some of its variations. In the end, it seems that Günther simply repeats for the Lewis-Stalnaker conditional what Priest suggested for a strict conditional. There might be something revealing in working with a Lewis-Stalnaker conditional instead of a strict one, but that is at least not made clear in (Günther 2022). It remains to be seen what are the particular implications when we combine the Lewis-Stalnaker conditional with Priest's framework.[4]

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[2] It can be seen as strongly connexive in some sense, however, if one is happy to spell out the notion of satisfiablity in a somewhat unusual manner (cf. (Omori &Wansing 2020, 514)).

[3] It should be noted that if one finds the approach via the relating semantics sufficiently intuitive, then there is an example of strongly connexive logics developed in (Jarmużek & Malinowski 2019). This, however, is not without problems either, but the details will go well beyond the aim of this note, and we will leave the discussion on this matter for another occasion.

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