

CONJUNCTION CLOSURE WITHOUT FACTIVITY: REASSESSING THE HYBRID PARADOX

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ABSTRACT: Francesco Praolini has recently put pressure on the view that justified believability is closed under conjunction introduction. Based on what he calls ‘the hybrid paradox,’ he argues that accepting the principle of conjunction closure for justified believability, quite surprisingly, entails that one must also accept the principle of factivity for justified believability, i.e. that there are no propositions that are justifiably believable and false at the same time. But proponents of conjunction closure can do without factivity, as I argue in this short note. A less demanding principle is available.

KEYWORDS: justified believability, conjunction closure, factivity, lottery paradox, preface paradox, hybrid paradox

It is a well-known fact among epistemologists that the following three individually plausible principles for justified believability, when taken together, give rise to paradoxes, the most popular being Kyburg’s lottery paradox or Makinson’s preface paradox:¹

Sufficiency. For any epistemic agent A , if a proposition p is very probable given A ’s evidence, then A is justified to believe p .

Conjunction Closure. For any epistemic agent A and any two propositions p and q , if A is justified to believe p at time t and A is justified to believe q at t , then A is also justified to believe their conjunction $p \& q$ at t .

No Contradictions. For any epistemic agent A , A is never justified to believe a logical contradiction, i.e. a proposition of the form $p \& \neg p$.

Recently, however, Francesco Praolini has argued that already *two* of these principles, namely Conjunction Closure and No Contradictions, lead to what he calls ‘the hybrid paradox,’ a new paradox sharing features of the lottery and the preface. Here is the set-up:

¹ See Henry E. Kyburg, *Probability and the Logic of Rational Belief* (Middletown: Wesleyan University Press 1961) and D. C. Makinson, “The Paradox of the Preface,” *Analysis* 25 (1965): 205–207.

Imagine that you have just completed a book that contains sentences that express all and only logically independent propositions that you are justified to believe. Because of that, *ex hypothesi*, for each sentence s_i in the body of the book, you are justified to believe that s_i is true. [...] Imagine, further, that you have submitted your manuscript to Perfectly Omniscient Press, and that its perfectly omniscient referee has reviewed it. Imagine that, following the policy of Perfectly Omniscient Press, the perfectly omniscient referee writes in his report that there is exactly one mistake in the book, without telling you, however, which claim is false. Assuming that you know that the referee of Perfectly Omniscient Press is perfectly omniscient, as soon as you read the referee report you come to know—and thereby justifiably believe—that there is exactly one mistake in the book. Given that you know—and justifiably believe—that there is exactly one mistake in the book, you are justified to believe that it is not the case that s_1 is true and s_2 is true ... and s_{n-1} is true and s_n is true.²

For brevity, let $J(p)$ state that p is justifiably believable for me. It then holds by assumption:

- (1) $J(s_1) \& \dots \& J(s_n)$

And iterated application of Conjunction Closure yields:

- (2) $J(s_1 \& \dots \& s_n)$

But by the referee report, it also seems to hold that:

- (3) $J(\neg[s_1 \& \dots \& s_n])$

And applying Conjunction Closure to (2) and (3) yields:

- (4) $J([s_1 \& \dots \& s_n] \& \neg[s_1 \& \dots \& s_n])$

Which violates No Contradictions. So, in the situation Praolini describes, Conjunction Closure and No Contradictions cannot be true together. Accordingly, to solve the paradox, we must either give up Conjunction Closure, No Contradictions or deny that the situation Praolini describes can possibly arise.

Since only few philosophers are willing to give up No Contradictions, Praolini argues that the most plausible strategy for proponents of Conjunction Closure to deny that the paradox can possibly arise is to reject (3) based on (1) and the following well-known, but quite demanding principle for justified believability:³

² Francesco Praolini, “No Justificatory Closure without Truth,” *Australasian Journal of Philosophy* 97 (2019): 720. For another recent paradox with a similar structure see the paradox of the pill due to Marvin Backes, “A Bitter Pill for Closure,” *Synthese* 196 (2019): 3773–3787) or the examples discussed in Clayton Littlejohn and Julien Dutant, “Justification, Knowledge, and Normality,” *Philosophical Studies* 177 (2019): 1593–1609.

³ A well-known exception is Priest, who *would* be willing to give up No Contradictions, see

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Factivity. For any epistemic agent A and any proposition p , if A is justified to believe that p , then p is true.

How does this strategy work? Praolini explains it as follows:

if Factivity is true, then no one can be justified to believe that the book contains a mistake. It is easy to understand why. Remember that the paradox discussed in this section asks us to imagine that you have written a book containing sentences that express all and only logically independent propositions that you are justified to believe. Because of this, *ex hypothesi*, you are justified to believe, of each of the claims s_i in your book, that s_i is true. Then, assuming that justified believability is factive, if you have justification for the truth of s_i , s_i must be true. Therefore, none of the claims in the book can be mistaken. For this reason, it also follows from Factivity that it is impossible to be justified to believe that the book contains a mistake.⁴

More formally, Factivity and (1) yield:

$$(5) \quad s_1 \& \dots \& s_n$$

Or equivalently:

$$(6) \quad \neg\neg(s_1 \& \dots \& s_n)$$

Then, by Factivity and *modus tollens*, we obtain:

$$(7) \quad \neg J(\neg[s_1 \& \dots \& s_n])$$

Which is the negation of (3). So, (1) and Factivity jointly refute (3).⁵ Accordingly, Praolini concludes that “the paradox shows that the acceptance of *Conjunction Closure* entails the acceptance of *Factivity*.”⁶

But Praolini’s conclusion is unnecessarily strong, if not false. For notice that his strategy only works because Factivity logically entails (but is not entailed by) the following principle which is already *sufficient* for the refutation of (3) based on (1) and which, presumably, proponents of Conjunction Closure will happily embrace:

Negation. For any epistemic agent A and propositions p_1 to p_n , if A is justified to believe that p_1 , A is justified to believe that p_2 , etc. and A is justified to believe that

Graham Priest, “What Is So Bad about Contradictions?,” *The Journal of Philosophy* 95 (1998): 410–26. It is worth mentioning that Praolini also considers but quickly dismisses other potential strategies for escaping the paradox. One of them is appealing to what Smith discusses as *Principle of Differential Defeat* in Martin Smith, “The Hardest Paradox for Closure,” *Erkenntnis* (2020), <https://doi.org/10.1007/s10670-020-00287-4>.

⁴ Praolini, “No Justificatory,” 724.

⁵ Refutation is defined as usual in terms of logical consequence: p refutes q if and only if p logically entails $\neg q$.

⁶ Praolini, “No Justificatory,” 724.

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p_n , then A is not justified to believe $\neg(p_1 \& \dots \& p_n)$.⁷

To see that Negation is in fact entailed by Factivity, assume that the latter holds while the former is false, i.e. assume that there are p_1 to p_n such that:

(8) $J(p_1) \& \dots \& J(p_n) \& J(\neg[p_1 \& \dots \& p_n])$

Then, applying Factivity to each conjunct, we obtain:

(9) $(p_1 \& \dots \& p_n) \& \neg(p_1 \& \dots \& p_n)$

Which is a logical contradiction. It is also easy to see that Negation is sufficient to refute (3) based on (1): simply apply Negation to (1) and the negation of (3) follows.

But if embracing Negation is enough to stop the hybrid paradox from arising, then there is no need for proponents of Conjunction Closure to embrace a principle as demanding as Factivity. They can simply embrace Negation instead—in fact, they *should*, if they also embrace No Contradictions, for Conjunction Closure and No Contradictions jointly *entail* Negation. To see this, assume that Conjunction Closure and No Contradictions are true while Negation is false, i.e. assume that there are p_1 to p_n such that:

(10) $J(p_1) \& \dots \& J(p_n) \& J(\neg[p_1 \& \dots \& p_n])$

Then, by multiple applications of Conjunction Closure, we get:

(11) $J([p_1 \& \dots \& p_n] \& \neg[p_1 \& \dots \& p_n])$

Which obviously violates No Contradictions. But if proponents of Conjunction Closure have a less demanding alternative to Factivity, then Praolini's claim that "the acceptance of Conjunction Closure surprisingly implies the acceptance of the thesis that justified believability is factive" is not true.⁸

There is, however, *some* truth in Praolini's claim. For Conjunction Closure does, together with the widely-accepted No Contradictions and a *further* principle for justified believability that might serve as a replacement for Sufficiency, entail

⁷ Negation can be seen as a generalization of principle D_J which figures in Rosenkranz's structural account of justification, see Sven Rosenkranz, "The Structure of Justification," *Mind* 127 (2018): 629–629. Loosely speaking, it states that if some proposition p is justifiably believable, then its negation $\neg p$ is not.

⁸ Praolini, "No Justificatory," 716. An anonymous referee raised the worry that embracing Negation instead of Factivity might lead to what Praolini calls 'maximally radical skepticism,' i.e. the view that one is not justified in believing *any* proposition. Praolini suggests that this is the case: "all other viable explanations imply radical scepticism" (724). However, I do not see how this would follow. After all, the reasoning presented here, just like Praolini's, starts with (1) as a premise in order to refute (3). Accordingly, the set of justifiably believable propositions is assumed to be *non-empty*.

Factivity. This principle can be considered a truth norm for justified believability and is obviously the converse of Factivity:⁹

Truth. For any epistemic agent A and any proposition p , if p is true, then A is justified to believe that p .

This principle might be appealing to both proponents and opponents of Sufficiency. Proponents might find it attractive because its basic idea is closely related to Sufficiency: if for them, being *very likely* true is already sufficient for justified believability, then *being* true should be sufficient on their view, too. Opponents might also be sympathetic to Truth if their reason for rejecting Sufficiency is that its antecedent is too weak and accordingly, that the standard for justified believability is too low. On their view, something stronger than high probability is required for justified believability. And this something could be the truth of the proposition in question.

Now, to see that No Contradictions, Conjunction Closure and Truth jointly entail Factivity, assume that the three former are true while the latter is false, i.e. assume that for some p it holds that:

$$(12) J(p) \ \& \ \neg p$$

Applying Truth to the second conjunct, we get:

$$(13) J(p) \ \& \ J(\neg p)$$

And by Conjunction Closure:

$$(14) J(p \ \& \ \neg p)$$

Which contradicts No Contradictions. Hence, No Contradictions, Conjunction Closure and Truth jointly entail Factivity.¹⁰

Time to summarize. Praolini has drawn our attention to an interesting new potential paradox for justified believability. But the conclusion he draws from it is unduly strong. There is nothing that forces proponents of Conjunction Closure to accept a principle as demanding as Factivity. In fact, a less demanding principle is available. And this principle should be very attractive to proponents of Conjunction

⁹ For instance, Boghossian discusses a version of this norm where justified believability is understood as epistemic permissibility, see Paul A. Boghossian, "The Normativity of Content," *Philosophical Issues* 13 (2003): 31–45.

¹⁰ This obviously entails that justified believability collapses to truth. Notice that there is more that can be said about the interconnections between the principles discussed in this note. For instance, Factivity not only entails Negation but also No Contradictions, and together with Truth it entails Conjunction Closure. Such details are, however, left for future research.

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Closure. Still, if proponents of Conjunction Closure also accept No Contradictions and Truth, then the acceptance of Factivity follows.¹¹

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