CONJUNCTION CLOSURE WITHOUT FACTIVITY: REASSESSING THE HYBRID PARADOX

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ABSTRACT: Francesco Praolini has recently put pressure on the view that justified believability is closed under conjunction introduction. Based on what he calls ‘the hybrid paradox,’ he argues that accepting the principle of conjunction closure for justified believability, quite surprisingly, entails that one must also accept the principle of factivity for justified believability, i.e. that there are no propositions that are justifiably believable and false at the same time. But proponents of conjunction closure can do without factivity, as I argue in this short note. A less demanding principle is available.

KEYWORDS: justified believability, conjunction closure, factivity, lottery paradox, preface paradox, hybrid paradox

It is a well-known fact among epistemologists that the following three individually plausible principles for justified believability, when taken together, give rise to paradoxes, the most popular being Kyburg’s lottery paradox or Makinson’s preface paradox:¹

- **Sufficiency.** For any epistemic agent A, if a proposition p is very probable given A’s evidence, then A is justified to believe p.

- **Conjunction Closure.** For any epistemic agent A and any two propositions p and q, if A is justified to believe p at time t and A is justified to believe q at t, then A is also justified to believe their conjunction p & q at t.

- **No Contradictions.** For any epistemic agent A, A is never justified to believe a logical contradiction, i.e. a proposition of the form p & ~p.

Recently, however, Francesco Praolini has argued that already two of these principles, namely Conjunction Closure and No Contradictions, lead to what he calls ‘the hybrid paradox,’ a new paradox sharing features of the lottery and the preface. Here is the set-up:

Imagine that you have just completed a book that contains sentences that express all and only logically independent propositions that you are justified to believe. Because of that, *ex hypothesi*, for each sentence $s_i$ in the body of the book, you are justified to believe that $s_i$ is true. [...] Imagine, further, that you have submitted your manuscript to Perfectly Omniscient Press, and that its perfectly omniscient referee has reviewed it. Imagine that, following the policy of Perfectly Omniscient Press, the perfectly omniscient referee writes in his report that there is exactly one mistake in the book, without telling you, however, which claim is false. Assuming that you know that the referee of Perfectly Omniscient Press is perfectly omniscient, as soon as you read the referee report you come to know—and thereby justifiably believe—that there is exactly one mistake in the book. Given that you know—and justifiably believe—that there is exactly one mistake in the book, you are justified to believe that it is not the case that $s_1$ is true and $s_2$ is true ... and $s_{n-1}$ is true and $s_n$ is true.$^2$

For brevity, let $J(p)$ state that $p$ is justifiably believable for me. It then holds by assumption:

$$ (1) \quad J(s_1) \land \ldots \land J(s_n) $$

And iterated application of Conjunction Closure yields:

$$ (2) \quad J(s_1 \land \ldots \land s_n) $$

But by the referee report, it also seems to hold that:

$$ (3) \quad J(\neg [s_1 \land \ldots \land s_n]) $$

And applying Conjunction Closure to (2) and (3) yields:

$$ (4) \quad J([s_1 \land \ldots \land s_n] \land \neg [s_1 \land \ldots \land s_n]) $$

Which violates No Contradictions. So, in the situation Praolini describes, Conjunction Closure and No Contradictions cannot be true together. Accordingly, to solve the paradox, we must either give up Conjunction Closure, No Contradictions or deny that the situation Praolini describes can possibly arise.

Since only few philosophers are willing to give up No Contradictions, Praolini argues that the most plausible strategy for proponents of Conjunction Closure to deny that the paradox can possibly arise is to reject (3) based on (1) and the following well-known, but quite demanding principle for justified believability:$^3$

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$^3$ A well-known exception is Priest, who *would* be willing to give up No Contradictions, see
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Factivity. For any epistemic agent \( A \) and any proposition \( p \), if \( A \) is justified to believe that \( p \), then \( p \) is true.

How does this strategy work? Praolini explains it as follows:

if Factivity is true, then no one can be justified to believe that the book contains a mistake. It is easy to understand why. Remember that the paradox discussed in this section asks us to imagine that you have written a book containing sentences that express all and only logically independent propositions that you are justified to believe. Because of this, \textit{ex hypothesi}, you are justified to believe, of each of the claims \( s \) in your book, that \( s \) is true. Then, assuming that justified believability is factive, if you have justification for the truth of \( s \), \( s \) must be true. Therefore, none of the claims in the book can be mistaken. For this reason, it also follows from Factivity that it is impossible to be justified to believe that the book contains a mistake.\(^4\)

More formally, Factivity and (1) yield:

\[
(5) \quad s_1 \land \ldots \land s_n
\]

Or equivalently:

\[
(6) \quad \neg (\neg s_1 \land \ldots \land \neg s_n)
\]

Then, by Factivity and \textit{modus tollens}, we obtain:

\[
(7) \quad \neg \neg (\neg [s_1 \land \ldots \land s_n])
\]

Which is the negation of (3). So, (1) and Factivity jointly refute (3).\(^5\) Accordingly, Praolini concludes that “the paradox shows that the acceptance of Conjunction Closure entails the acceptance of Factivity.”\(^6\)

But Praolini’s conclusion is unnecessarily strong, if not false. For notice that his strategy only works because Factivity logically entails (but is not entailed by) the following principle which is already \textit{sufficient} for the refutation of (3) based on (1) and which, presumably, proponents of Conjunction Closure will happily embrace:

Negation. For any epistemic agent \( A \) and propositions \( p_1 \) to \( p_n \), if \( A \) is justified to believe that \( p_1 \), \( A \) is justified to believe that \( p_2 \), etc. and \( A \) is justified to believe that

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\(^4\) Praolini, “No Justificatory,” 724.

\(^5\) Refutation is defined as usual in terms of logical consequence: \( p \) refutes \( q \) if and only if \( p \) logically entails \( \neg q \).

\(^6\) Praolini, “No Justificatory,” 724.
To see that Negation is in fact entailed by Factivity, assume that the latter holds while the former is false, i.e. assume that there are \( p_1 \) to \( p_n \) such that:

\[
(8) \ J(p_1) \ & \ ... \ & J(p_n) \ & J(\neg[p_1 \ & \ ... \ & p_n])
\]

Then, applying Factivity to each conjunct, we obtain:

\[
(9) \ (p_1 \ & \ ... \ & p_n) \ & \ \neg(p_1 \ & \ ... \ & p_n)
\]

Which is a logical contradiction. It is also easy to see that Negation is sufficient to refute (3) based on (1): simply apply Negation to (1) and the negation of (3) follows.

But if embracing Negation is enough to stop the hybrid paradox from arising, then there is no need for proponents of Conjunction Closure to embrace a principle as demanding as Factivity. They can simply embrace Negation instead—in fact, they should, if they also embrace No Contradictions, for Conjunction Closure and No Contradictions jointly entail Negation. To see this, assume that Conjunction Closure and No Contradictions are true while Negation is false, i.e. assume that there are \( p_i \) to \( p_n \) such that:

\[
(10) \ J(p_1) \ & \ ... \ & J(p_n) \ & J(\neg[p_1 \ & \ ... \ & p_n])
\]

Then, by multiple applications of Conjunction Closure, we get:

\[
(11) \ J([p_1 \ & \ ... \ & p_n] \ & \ \neg[p_1 \ & \ ... \ & p_n])
\]

Which obviously violates No Contradictions. But if proponents of Conjunction Closure have a less demanding alternative to Factivity, then Praolini’s claim that “the acceptance of Conjunction Closure surprisingly implies the acceptance of the thesis that justified believability is factive” is not true.\(^\text{8}\)

There is, however, some truth in Praolini’s claim. For Conjunction Closure does, together with the widely-accepted No Contradictions and a further principle for justified believability that might serve as a replacement for Sufficiency, entail

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\(^\text{7}\) Negation can be seen as a generalization of principle D, which figures in Rosenkranz’s structural account of justification, see Sven Rosenkranz, “The Structure of Justification,” *Mind* 127 (2018): 629–629. Loosely speaking, it states that if some proposition \( p \) is justifiably believable, then its negation \( \neg p \) is not.

\(^\text{8}\) Praolini, “No Justificatory,” 716. An anonymous referee raised the worry that embracing Negation instead of Factivity might lead to what Praolini calls ‘maximally radical skepticism,’ i.e. the view that one is not justified in believing any proposition. Praolini suggests that this is the case: “all other viable explanations imply radical scepticism” (724). However, I do not see how this would follow. After all, the reasoning presented here, just like Praolini’s, starts with (1) as a premise in order to refute (3). Accordingly, the set of justifiably believable propositions is assumed to be non-empty.
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Factivity. This principle can be considered a truth norm for justified believability and is obviously the converse of Factivity:\(^9\)

Truth. For any epistemic agent \(A\) and any proposition \(p\), if \(p\) is true, then \(A\) is justified to believe that \(p\).

This principle might be appealing to both proponents and opponents of Sufficiency. Proponents might find it attractive because its basic idea is closely related to Sufficiency: if for them, being very likely true is already sufficient for justified believability, then being true should be sufficient on their view, too. Opponents might also be sympathetic to Truth if their reason for rejecting Sufficiency is that its antecedent is too weak and accordingly, that the standard for justified believability is too low. On their view, something stronger than high probability is required for justified believability. And this something could be the truth of the proposition in question.

Now, to see that No Contradictions, Conjunction Closure and Truth jointly entail Factivity, assume that the three former are true while the latter is false, i.e. assume that for some \(p\) it holds that:

\[(12) \ J(p) & \neg p\]

Applying Truth to the second conjunct, we get:

\[(13) \ J(p) & J(\neg p)\]

And by Conjunction Closure:

\[(14) \ J(p & \neg p)\]

Which contradicts No Contradictions. Hence, No Contradictions, Conjunction Closure and Truth jointly entail Factivity.\(^{10}\)

Time to summarize. Praolini has drawn our attention to an interesting new potential paradox for justified believability. But the conclusion he draws from it is unduly strong. There is nothing that forces proponents of Conjunction Closure to accept a principle as demanding as Factivity. In fact, a less demanding principle is available. And this principle should be very attractive to proponents of Conjunction Closure.

\(^9\) For instance, Boghossian discusses a version of this norm where justified believability is understood as epistemic permissibility, see Paul A. Boghossian, “The Normativity of Content,” \textit{Philosophical Issues} 13 (2003): 31–45.

\(^{10}\) This obviously entails that justified believability collapses to truth. Notice that there is more that can be said about the interconnections between the principles discussed in this note. For instance, Factivity not only entails Negation but also No Contradictions, and together with Truth it entails Conjunction Closure. Such details are, however, left for future research.
Closure. Still, if proponents of Conjunction Closure also accept No Contradictions and Truth, then the acceptance of Factivity follows.\footnote{I would like to thank two anonymous referees and Roman Heil for a number of helpful comments and a vivid discussion. This work was funded by the University of Hamburg as part of the Excellence Strategy and by the German Research Foundation (DFG) as part of grant SCHU 3080/3-1 to Moritz Schulz.}