THE APORIA OF OMNISCIENCE

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ABSTRACT This paper introduces a new aporia, the aporia of omniscience. The puzzle consists of three propositions: (1) It is possible that there is someone who is necessarily omniscient and infallible, (2) It is necessary that all beliefs are historically settled, and (3) It is possible that the future is open. Every sentence in this set is intuitively reasonable and there are prima facie plausible arguments for each of them. However, the whole set {(1), (2), (3)} is inconsistent. Therefore, it seems to be that case that at least one of the propositions in this set must be false. I discuss some possible solutions to the problem and consider some arguments for and against these solutions.

KEYWORDS: aporia, omniscience, infallibility, historically settled beliefs, the open future

1. Introduction

In this paper, I will discuss a new aporia, the aporia of omniscience. This puzzle includes ideas about omniscience, infallibility, the modal status of beliefs and the open future. So, maybe a more comprehensive name would be 'the aporia of omniscience, infallibility, historically settled beliefs and the open future.' But this is too long. So, I will call the puzzle 'the aporia of omniscience,' since the concept of omniscience plays an essential role in the problem. The aporia of omniscience consists of the following three propositions:

(1) It is possible that there is someone who is necessarily omniscient and infallible.

(2) It is necessary that all beliefs are historically settled.

(3) It is possible that the future is open. That is, it is possible that there is a proposition A such that it will (some time in the future) be the case that A even though it is not historically settled that it will (some time in the future) be the case that A.

Each proposition in {(1), (2), (3)} is intuitively plausible, but the whole set entails a contradiction. Hence, together these sentences constitute an aporia. The aporia of omniscience should be interesting to anyone who is concerned about such topics as the nature and possibility of omniscience and infallibility, the modal status of beliefs, and the nature of the future (whether it is open or not).[1]

[1] The concept of omniscience has been discussed in the philosophy of religion for a long time. See, for example, George I. Mavrodes, "Omniscience," in *A Companion to Philosophy of*

I have asserted that {(1), (2), (3)} is inconsistent but that the propositions in this set are intuitively plausible. To justify this claim, I will first say a few more words about (1)–(3) and express these sentences in symbols. '◆A' reads as 'It is possible that A'; '■A' reads as 'It is necessary that A'; '□A' reads as 'It is historically settled (necessary) that A'; '◇A' reads as 'It is historically possible that A'; 'Bc' reads as 'Individual c believes that A'; 'Kc' reads as 'Individual c knows that A'; 'FA' reads as 'It will some time in the future be the case that A'; 'Oc' reads as 'Individual c is omniscient' ('O' is a predicate) and 'Ic' reads as 'Individual c is infallible' ('I' is a predicate). All other symbols are interpreted as usual. (1)–(3) can now be symbolized in the following way:

(1) ◆∃x(■Ox∧Ix). It is possible that there is some (individual) x such that it is necessary that x is omniscient and x is infallible.

Note that (1) only says that it is *possible* that there is an individual of a certain kind. (1) is consistent with the proposition that there are no (existing) individuals of this sort. If there is an (existing) individual that is necessarily omniscient and infallible, then obviously it is possible that there is an individual of this kind. But the converse does not necessarily hold. We shall define the concepts of omniscience and infallibility in the following way:

(O) ■∀x(Ox↔∀A(A→KxA)). It is necessary that for every (individual) x: x is omniscient if and only if (iff) for every (proposition) A, if A (is true) then x knows that A.

(I) ■∀x(Ix↔■∀A(BxA→A)). It is necessary that for every (individual) x: x is

Religion, Second Edition, eds. Charles Taliaferro, Paul Draper and Philip L. Quinn (Wiley-Blackwell, 2010), 251–257, Edward R. Wierenga, "Omniscience," in *The Oxford Handbook of Philosophical Theology*, eds. Thomas P. Flint and Michael C. Rea (Oxford and New York: Oxford University Press, 2009), 129–144 and Paul Weingartner, *Omniscience From a Logical Point of View* (Frankfurt: Ontos Verlag, 2008) for more on this notion and many relevant references. Similar puzzles, which concern the compatibility of God's foreknowledge and human free will, have also been discussed in the philosophy of religion; see, for example, William Lane Craig, *The Problem of Divine Foreknowledge and Future Contingents from Aristotle to Suarez* (Leiden: E. J. Brill, 1988), William Hasker, "Divine Knowledge and Human Freedom," in *The Oxford Handbook of Free Will*, ed. Robert Kane (Oxford: Oxford University Press, 2011), 39–53 and Linda Zagzebski, "Omniscience, Time, and Freedom," in *The Blackwell Guide to the Philosophy of Religion*, ed. William E. Mann (Blackwell Publishing, 2005), 3–25. However, there are important differences between such puzzles and the aporia of omniscience. First, I am not concerned with any specific religious doctrines in this paper even though the arguments might be relevant for several discussions within the philosophy of religion. Second, the aporia does not involve any claims about our free will. Third, the details of the arguments in this paper are quite different from the details of similar arguments that can be found in the literature.

infallible iff it is necessary that for every (proposition) A, if x believes that A then A.

(O) is a definition of what we mean by 'omniscient,' and (I) is a definition of what we mean by 'infallible.' Obviously, (O) [(I)] in itself does not entail that there is anyone who is omniscient [infallible]. The first quantifier in (O) [I] varies over individuals and the second over propositions or sentences. Note that (I) entails that it is *necessary* that no infallible individual has any false beliefs, while (O) does not entail that an omniscient individual necessarily knows every truth.

(2) $\blacksquare\forall x\forall A(BxA\rightarrow\Box BxA)$. It is necessary that for every (individual) x and for every (proposition) A, if x believes that A, then it is historically settled that x believes that A.

Note that '$\Box BxA$' in (2) does not assert that it is *necessary* that x believes that A; it says that it is *historically settled* that x believes that A. This is prima facie plausible. Facts about what someone believes seem to be historically settled; if someone believes something, it appears to be historically impossible for her not to believe this thing. Suppose that (2) is false. Then it is possible that there is someone who believes something even though it is not historically settled that she believes it. This is counterintuitive. Again, note that the first quantifier in (2) varies over individuals while the second varies over propositions or sentences.

(3) $\blacklozenge\exists A(\mathsf{F}A\wedge\neg\Box\mathsf{F}A)$. It is possible that there is a proposition A such that it will (some time in the future) be the case that A even though it is not historically necessary that A.

(3) is one way of expressing the idea that the future can be open. Suppose that (3) is not true. Then $\blacksquare\forall A(\mathsf{F}A\rightarrow\Box\mathsf{F}A)$ is true; that is, then it is necessary that for every (proposition) A, if it will be the case that A then it is historically settled that it will be the case that A. If (3) is false, then it is necessary that nothing that will happen is such that it is historically possible that it will not happen. In other words, then it is necessary that the future is not open. The idea that the future is open can be symbolized in the following way: $\exists A(\mathsf{F}A\wedge\neg\Box\mathsf{F}A)$. Note that (3) only says that it is *possible* that the future is open. (3) is consistent both with the proposition that the future is *not* open and with the proposition that the future *is* open.

'$\blacksquare$' and '$\blacklozenge$' are used as 'absolute' S5-operators in this paper, and '$\Box$' and '$\Diamond$' are used as 'relative' S5-operators. $\blacksquare A$ is true in a possible world at a moment in time iff A is true in every possible world at every moment in time. $\blacklozenge A$ is true in a possible world at a moment in time iff A is true in some possible world at some moment in time. $\Box A$ is true in a possible world w at a moment in time t iff A is true in every possible world that is still historically accessible from w at t. $\Diamond A$ is true in a possible world w at a moment in time t iff A is true in some possible world that is still historically accessible from w at t. Intuitively, this means that □A is true in a possible world at a moment in time iff A is true no matter how the future turns out, and ◇A is true in a possible world at a moment in time iff there is still some way in which the future might evolve that would lead to the truth of A. Alternatively, we can say that A is historically possible in a possible world w at a certain moment in time t iff A is still possible at t given the history of w and the laws of nature that hold in w, and it is historically necessary that A in w at t iff A is true at t in every possible world that is still possible at t given the history of w and the laws of nature that hold in w. ■A is stronger than □A and ◆A is weaker than ◇A.[2] KcA is true in w at t iff A is true in all possible worlds that are epistemically accessible from w at t for the individual c. BcA is true in w at t iff A is true in all possible worlds that are doxastically accessible from w at t for the individual c.[3] The truth-conditions for the other formulas are standard.[4]

The main argument

Before I turn to the main argument and show that {(1), (2), (3)} is inconsistent, I will establish a lemma called '(O′).' Intuitively, (O′) says that every necessarily omniscient individual necessarily believes every truth. '¬■E,' '¬∀E,' '■E,' '∀E,' 'MP' ('Modus Ponens'), '◆E,' '∃E,' '¬□E' etc. in the derivations below represent standard derivation rules in propositional logic, predicate logic and modal logic. 'E' is an abbreviation of 'elimination.' 'PL' means that the step follows by standard propositional principles.

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[2] For more on modal logic, see, for example, Patrick Blackburn, Maarten De Rijke, Yde Venema, *Modal Logic* (Cambridge: Cambridge University Press, 2001), Brian F. Chellas, *Modal Logic: An Introduction* (Cambridge: Cambridge University Press, 1980) and George Edward Hughes and Max John Cresswell, *An Introduction to Modal Logic* (London: Routledge, 1968 (Reprinted 1990)).

[3] For more on epistemic and doxastic logic, see, for example, Roland Fagin, Joseph Y. Halpern, Yoram Moses, and Moshe Y. Vardi, *Reasoning About Knowledge* (Cambridge, Mass., London, England: The MIT Press, 1995) and John-Jules Ch. Meyer, and Wiebe van der Hoek, *Epistemic Logic for AI and Computer Science* (Cambridge University Press, 1995).

[4] Many references to the relevant literature on temporal logic can be found in John P. Burgess, "Basic Tense Logic," in *Handbook of Philosophical Logic*, Vol. 2, eds. Dov M. Gabbay and Franz Guenthner (Dordrecht: Reidel, 1984), 89–133 and Peter Øhrstrøm and Per Frederik Vilhelm Hasle, *Temporal Logic: From Ancient Ideas to Artificial Intelligence* (Dordrecht/Boston/London: Kluwer Academic Publishers, 1995). For some ideas about how to combine modal logic and tense logic, see Richmond H. Thomason, "Combinations of Tense and Modality," in *Handbook of Philosophical Logic*, vol. 2, eds. Dov M. Gabbay and Franz Guenthner (Dordrecht: Reidel, 2002), 135–165, (2nd edition 7, 2002, 205–234).

(O′) $\blacksquare\forall x(\blacksquare Ox\rightarrow\blacksquare\forall A(A\rightarrow BxA))$. It is necessary that every (individual) x who is necessarily omniscient is such that it is necessary that for every (proposition) A if A is the case then x believes that A.

In the derivation of (O′), I will use a basic assumption (KB). In a slogan, (KB) says that knowledge entails belief. Most epistemologists accept this proposition.

(KB) $\blacksquare\forall x\forall A(KxA\rightarrow BxA)$. It is necessary that for every (individual) x and for every (proposition) A, if x knows that A then x believes that A.

Let us now show that (O) and (KB) entail (O′). To establish this we assume that (O) and (KB) are true in some possible world w_0 at some moment in time t_0 and that (O′) is false in w_0 at t_0. This leads to a contradiction. Hence, (O′) follows from (O) and (KB).

(1) $\neg\blacksquare\forall x(\blacksquare Ox\rightarrow\blacksquare\forall A(A\rightarrow BxA)), w_0t_0$	[Assumption]
(2) $\neg\forall x(\blacksquare Ox\rightarrow\blacksquare\forall A(A\rightarrow BxA)), w_1t_1$	[1, $\neg\blacksquare$E]
(3) $\neg(\blacksquare Oc\rightarrow\blacksquare\forall A(A\rightarrow BcA)), w_1t_1$	[2, $\neg\forall$E]
(4) $\blacksquare Oc, w_1t_1$	[3, PL]
(5) $\neg\blacksquare\forall A(A\rightarrow BcA), w_1t_1$	[3, PL]
(6) $\neg\forall A(A\rightarrow BcA), w_2t_2$	[5, $\neg\blacksquare$E]
(7) $\neg(X\rightarrow BcX), w_2t_2$	[6, $\neg\forall$E]
(8) X, w_2t_2	[7, PL]
(9) $\neg BcX, w_2t_2$	[7, PL]
(10) $\forall x(Ox\leftrightarrow\forall A(A\rightarrow KxA)), w_2t_2$	[(O), $\blacksquare$E]
(11) $Oc\leftrightarrow\forall A(A\rightarrow KcA), w_2t_2$	[10, $\forall$E]
(12) Oc, w_2t_2	[4, $\blacksquare$E]
(13) $\forall A(A\rightarrow KcA), w_2t_2$	[11, 12, PL]
(14) $X\rightarrow KcX, w_2t_2$	[13, $\forall$E]
(15) KcX, w_2t_2	[8, 14, MP]
(16) $\forall x\forall A(KxA\rightarrow BxA), w_2t_2$	[(KB), $\blacksquare$E]
(17) $\forall A(KcA\rightarrow BcA), w_2t_2$	[16, $\forall$E]
(18) $KcX\rightarrow BcX, w_2t_2$	[17, $\forall$E]
(19) BcX, w_2t_2	[15, 18, MP]
(20) $BcX\wedge\neg BcX, w_2t_2$	[19, 9, PL]

We are now in a position to establish that {(1), (2), (3)} is inconsistent. I will assume that all sentences in {(1), (2), (3)} as well as (I) and (O) are true in a possible world w_0 at a moment in time t_0 and derive a contradiction. Intuitively, '$w_3 \equiv_{t_1} w_1$' in the derivation below reads as 'the possible world w_3 is historically accessible from the possible world w_1 at the time t_1,' and 'A, w_0t_0' reads as 'A is true in the possible world w_0 at the time t_0,' etc. The following deduction shows that {(1), (2), (3)} is inconsistent. Let us call this argument 'the main argument.'

(1) $\exists A(\mathsf{F}A \wedge \neg\Box\mathsf{F}A)$, w_1t_1	[(3), $\blacklozenge$E]
(2) $\mathsf{F}X \wedge \neg\Box\mathsf{F}X$, w_1t_1	[1, $\exists$E]
(3) $\mathsf{F}X$, w_1t_1	[3, PL]
(4) $\neg\Box\mathsf{F}X$, w_1t_1	[3, PL]
(5) $\exists x(\blacksquare Ox \wedge Ix)$, w_2t_2	[(1), $\blacklozenge$E]
(6) $\blacksquare Oc \wedge Ic$, w_2t_2	[5, $\exists$E]
(7) $\blacksquare Oc$, w_2t_2	[6, PL]
(8) Ic, w_2t_2	[6, PL]
(9) $\forall x(Ix \leftrightarrow \blacksquare\forall A(BxA \rightarrow A))$, w_2t_2	[(I), $\blacksquare$E]
(10) $Ic \leftrightarrow \blacksquare\forall A(BcA \rightarrow A)$, w_2t_2	[9, $\forall$E]
(11) $\blacksquare\forall A(BcA \rightarrow A)$, w_2t_2	[8, 10, PL]
(12) $\forall x(\blacksquare Ox \rightarrow \blacksquare\forall A(A \rightarrow BxA))$, w_2t_2	[(O′), $\blacksquare$E]
(13) $\blacksquare Oc \rightarrow \blacksquare\forall A(A \rightarrow BcA)$, w_2t_2	[12, $\forall$E]
(14) $\blacksquare\forall A(A \rightarrow BcA)$, w_2t_2	[7, 13, MP]
(15) $\forall A(A \rightarrow BxA)$, w_1t_1	[14, $\blacksquare$E]
(16) $\mathsf{F}X \rightarrow Bx\mathsf{F}X$, w_1t_1	[15, $\forall$E]
(17) $Bx\mathsf{F}X$, w_1t_1	[3, 16, MP]
(18) $\forall x\forall A(BxA \rightarrow \Box BxA)$, w_1t_1	[(2), $\blacksquare$E]
(19) $\forall A(BcA \rightarrow \Box BcA)$, w_1t_1	[18, $\forall$E]
(20) $Bc\mathsf{F}X \rightarrow \Box Bc\mathsf{F}X$, w_1t_1	[19, $\forall$E]
(21) $\Box Bc\mathsf{F}X$, w_1t_1	[17, 20, MP]
(22) $w_3 \equiv_{t_1} w_1$	[4, $\neg\Box$E]
(23) $\neg\mathsf{F}X$, w_3t_1	[4, $\neg\Box$E]
(24) $Bc\mathsf{F}X$, w_3t_1	[21, 22, $\Box$E]

(25) $\forall A(BcA \rightarrow A)$, w_3t_1 [11, ■E]

(26) $BcFX \rightarrow FX$, w_3t_1 [25, $\forall$E]

(27) FX, w_3t_1 [24, 26, MP]

(28) $FX \wedge \neg FX$, w_3t_1 [27, 23, PL]

Obviously, (28) is a contradiction. Accordingly, {(1), (2), (3)} entails a contradiction. Therefore {(1), (2), (3)} is inconsistent. It follows that at least one sentence in {(1), (2), (3)} must be false. This fact justifies the assertion that {(1), (2), (3)} is an aporia.

2. Arguments for the Sentences in the Aporia

I have said that the sentences in the aporia of omniscience are intuitively reasonable. However, a sceptic might argue that we cannot rely simply on our intuitions. To take the puzzle seriously we also need some independent reasons for the sentences in {(1), (2), (3)}. Therefore, I will consider some arguments for the sentences in the aporia in this section.

Arguments for (1): It is possible that there is someone who is necessarily omniscient and infallible

Is it possible that there is someone who is necessarily omniscient and infallible? I will consider two arguments for this proposition, which I will call the 'the argument from conceivability' and 'the argument from doxastic consistency.'

The argument from conceivability

> (CP) It is conceivable that there is someone who is necessarily omniscient and infallible.

Hence,

> (1) It is possible that there is someone who is necessarily omniscient and infallible.

If conceivability implies possibility, as some seem to think, then the argument from conceivability is valid. Furthermore, if (CP) is true, as it certainly seems to be, then the argument is also sound and we must conclude that (1) is true. However, the conceivability implies possibility thesis is controversial. It is not obvious that everything that is conceivable is possible. Nevertheless, it appears to be reasonable to say that conceivability 'indicates' possibility. If something is conceivable, then we have a prima facie reason to believe that it is possible, even if it should turn out to be the case that there are no necessary connections between conceivability and possibility. Accordingly, the argument from conceivability seems to give some support to (1).[5] The following scenario underpins (CP). Imagine an individual called 'The All-knowing One.' Suppose that The All-knowing One knows every truth in every possible world and that it is necessary that everything The All-knowing One believes is true. Then The All-knowing One is both necessarily omniscient and infallible. Such a being is conceivable. Hence, it is conceivable that there is an individual who is necessarily omniscient and infallible. Furthermore, this scenario does not seem to entail any contradiction. Therefore, (CP) is true. It follows that we have a prima facie reason to believe that (1) is true.

The argument from doxastic consistency

Here is a different argument for (1), which we will call 'the argument from doxastic consistency.' (1) follows from the proposition that it is possible that there is someone who is necessarily omniscient and necessarily doxastically consistent; in symbols, ◆∃x(■Ox∧Ix) follows from (1′) = ◆∃x(■Ox∧■Dx). An individual x is doxastically consistent iff there is no (proposition) A such that x believes that A and x believes that not-A. In other words, doxastic consistency is defined in the following way:

> (DC) ■∀x(Dx↔¬∃A(BxA∧Bx¬A)). It is necessary that an individual x is doxastically consistent iff there is no A such that x believes that A and x believes that not-A.

This argument is interesting since doxastic consistency seems to be a weaker property than infallibility. If someone is infallible, she is doxastically consistent, but someone can be (necessarily) doxastically consistent without being infallible. So, the fact that ◆∃x(■Ox∧■Dx) entails ◆∃x(■Ox∧Ix) is noteworthy. To prove this proposition, we will first establish that everyone who is necessarily omniscient and necessarily doxastically consistent is infallible given that knowledge entails belief. Let us call this lemma (ODI).

> (ODI) ■∀x((■Ox∧■Dx)→Ix). It is necessary that every (individual) x who is necessarily omniscient and necessarily doxastically consistent is infallible.

Here is the proof of (ODI). (The proof is a reductio argument where we assume the negation of (ODI) and derive a contradiction. This establishes our result.)

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[5] For more on the conceivability implies possibility thesis and for some arguments for and against this principle, see, for example, Tamar Szabó Gendler and John Hawthorne (eds.), *Conceivability and Possibility* (Oxford: Clarendon Press, 2002).

(1) $\neg\blacksquare\forall x((\blacksquare Ox\wedge\blacksquare Dx)\rightarrow Ix)$, w_0t_0	[Assumption]
(2) $\neg\forall x((\blacksquare Ox\wedge\blacksquare Dx)\rightarrow Ix)$, w_1t_1	[1, $\neg\blacksquare$E]
(3) $\neg((\blacksquare Oc\wedge\blacksquare Dc)\rightarrow Ic)$, w_1t_1	[2, $\neg\forall$E]
(4) $\blacksquare Oc\wedge\blacksquare Dc$, w_1t_1	[3, PL]
(5) $\neg Ic$, w_1t_1	[3, PL]
(6) $\blacksquare Oc$, w_1t_1	[4, PL]
(7) $\blacksquare Dc$, w_1t_1	[4, PL]
(8) $\forall x(Ix\leftrightarrow\blacksquare\forall A(BxA\rightarrow A))$, w_1t_1	[(I), $\blacksquare$E]
(9) $Ic\leftrightarrow\blacksquare\forall A(BcA\rightarrow A)$, w_1t_1	[8, $\forall$E]
(10) $\neg\blacksquare\forall A(BcA\rightarrow A)$, w_1t_1	[5, 9, PL]
(11) $\neg\forall A(BcA\rightarrow A)$, w_2t_2	[10, $\neg\blacksquare$E]
(12) $\neg(BcX\rightarrow X)$, w_2t_2	[11, $\neg\forall$E]
(13) BcX, w_2t_2	[12, PL]
(14) $\neg X$, w_2t_2	[12, PL]
(15) $\forall x(Ox\leftrightarrow\forall A(A\rightarrow KxA))$, w_2t_2	[(O), $\blacksquare$E]
(16) $Oc\leftrightarrow\forall A(A\rightarrow KcA)$, w_2t_2	[15, $\forall$E]
(17) Oc, w_2t_2	[6, $\blacksquare$E]
(18) $\forall A(A\rightarrow KcA)$, w_2t_2	[16, 17, PL]
(19) $\neg X\rightarrow Kc\neg X$, w_2t_2	[18, $\forall$E]
(20) $\forall x(Dx\leftrightarrow\neg\exists A(BxA\wedge Bx\neg A))$, w_2t_2	[(DC), $\blacksquare$E]
(21) $Dc\leftrightarrow\neg\exists A(BcA\wedge Bc\neg A)$, w_2t_2	[20, $\forall$E]
(22) Dc, w_2t_2	[7, $\blacksquare$E]
(23) $\neg\exists A(BcA\wedge Bc\neg A)$, w_2t_2	[21, 22, PL]
(24) $\neg(BcX\wedge Bc\neg X)$, w_2t_2	[23, $\neg\exists$E]
(25) $\forall x\forall A(KxA\rightarrow BxA)$, w_2t_2	[(KB), $\blacksquare$E]
(26) $\forall A(KcA\rightarrow BcA)$, w_2t_2	[25, $\forall$E]
(27) $Kc\neg X\rightarrow Bc\neg X$, w_2t_2	[26, $\forall$E]
(28) $Kc\neg X$, w_2t_2	[14, 19, MP]
(29) $Bc\neg X$, w_2t_2	[27, 28, MP]
(30) $BcX\wedge Bc\neg X$, w_2t_2	[13, 29, PL]

(31) $(BcX \wedge Bc\neg X) \wedge \neg(BcX \wedge Bc\neg X)$, w_2t_2 [30, 24, PL]

Clearly (31) is a contradiction. Hence, the assumption that (ODI) is false in some possible world at some moment in time must be false. It follows that (ODI) is valid. We are now in a position to establish that $\blacklozenge\exists x(\blacksquare Ox \wedge Ix)$ follows from $\blacklozenge\exists x(\blacksquare Ox \wedge \blacksquare Dx)$. Here is the proof:

(1) $\blacklozenge\exists x(\blacksquare Ox \wedge \blacksquare Dx)$, w_0t_0 [Assumption]

(2) $\neg\blacklozenge\exists x(\blacksquare Ox \wedge Ix)$, w_0t_0 [Assumption]

(3) $\exists x(\blacksquare Ox \wedge \blacksquare Dx)$, w_1t_1 [1, $\blacklozenge$E]

(4) $\blacksquare Oc \wedge \blacksquare Dc$, w_1t_1 [3, $\exists$E]

(5) $\blacksquare Oc$, w_1t_1 [4, PL]

(6) $\blacksquare Dc$, w_1t_1 [4, PL]

(7) $\forall x((\blacksquare Ox \wedge \blacksquare Dx) \rightarrow Ix)$, w_1t_1 [(ODI), $\blacksquare$E]

(8) $(\blacksquare Oc \wedge \blacksquare Dc) \rightarrow Ic$, w_1t_1 [7, $\forall$E]

(9) Ic, w_1t_1 [4, 8, MP]

(10) $\neg\exists x(\blacksquare Ox \wedge Ix)$, w_1t_1 [2, $\neg\blacklozenge$E]

(11) $\neg(\blacksquare Oc \wedge Ic)$, w_1t_1 [10, $\neg\exists$E]

(12) $\blacksquare Oc \wedge Ic$, w_1t_1 [5, 9, PL]

(13) $(\blacksquare Oc \wedge Ic) \wedge \neg(\blacksquare Oc \wedge Ic)$, w_1t_1 [12, 11, PL]

Arguments for (2): It is necessary that all beliefs are historically settled

Is it true that it is necessary that all beliefs are historically settled? In this section, I will consider an ‘intuitive’ argument for proposition (2) and a semantic argument.

The intuitive argument

Suppose that (2) is not true. Then it seems that what someone believes might depend on what will turn out to be the case in the future. This is counterintuitive. Consider the following example. It is not historically settled that a democrat will win the next election. It is historically possible that a democrat will win and it is historically possible that it is not the case that a democrat will win. Moreover, suppose that The Omniscient One is necessarily omniscient and infallible. Then whether The Omniscient One believes that a democrat will win or not is not yet historically settled. If a democrat will win, then it will be true once the democrat has won that it was the case (now) that The Omniscient One believed that a democrat would win; and if a democrat will not win, then it will be true once the democrat has lost that it was the case (now) that The Omniscient One did not believe that a democrat would win. However, as of this moment it is not a settled fact whether The Omniscient One believes that a democrat will win or not. Scenarios such as this clearly suggest that (2) is true.

Furthermore, this is not the only possible reason for (2). There are several possible semantic arguments for this proposition. Let us consider one such argument, which I will call 'the argument from semantics.'

The argument from semantics

It is possible to show that $\blacksquare\forall x\forall A(BxA\rightarrow\Box BxA)$ is valid if we assume the following semantic condition:

> (SC) If a possible world w′ is historically accessible from a possible world w at a time t and a possible world w″ is doxastically accessible from w′ at t for an individual c, then w″ is doxastically accessible from w at t for c.

Suppose that (SC) holds. Then we can prove that (2) is valid. Assume that (2) = $\blacksquare\forall x\forall A(BxA\rightarrow\Box BxA)$ is not true in the possible world w_0 at the moment in time t_0. Then there is some possible world w_1 and some moment of time t_1, such that $\forall x\forall A(BxA\rightarrow\Box BxA)$ is false in w_1 at t_1. Accordingly, $\forall A(BcA\rightarrow\Box BcA)$ is false in w_1 at t_1 (for some arbitrary individual c). Consequently, $BcX\rightarrow\Box BcX$ is false in w_1 at t_1 (where 'X' represents some arbitrary proposition). Therefore, BcX is true in w_1 at t_1, while $\Box BcX$ is false in w_1 at t_1. It follows that there is a possible world w_2 that is historically accessible from w_1 at t_1 in which BcX is false at t_1, for $\Box BcX$ is false in w_1 at t_1. Accordingly, there is some possible world w_3 that is doxastically accessible from w_2 at t_1 for c in which X is false at t_1. Since w_2 is historically accessible from w_1 at t_1 and w_3 is doxastically accessible from w_2 at t_1 for c, w_3 is doxastically accessible from w_1 at t_1 for c (condition (SC)). In conclusion, X is true in w_3 at t_1, for BcX is true in w_1 at t_1 and w_3 is doxastically accessible from w_1 at t_1 for c. Yet, this is contradictory. So, our assumption is false. It follows that $\blacksquare\forall x\forall A(BxA\rightarrow\Box BxA)$ is valid.

Arguments for 3: It is possible that the future is open

Is it true that it is possible that the future is open? We have seen that there are arguments for (1) and (2). I will now consider two brief arguments for (3), which I will call 'the argument from science' and 'the argument from conceivability.'

The argument from science

According to the argument from science, it is a scientific fact that the future is open. It is a scientific fact because according to the dominating interpretations of modern physics at least some processes in nature are indeterminate. It is, for example, historically possible that the photon will pass through the right slit and it is historically possible that it will pass through the left slit. Hence, the future is open. And if the future *is* open, then it is obvious that it is *possible* that it is open.[6]

The argument from conceivability

Suppose (contrary-to-the-facts?) that the future is not open. It is still *conceivable* that the future is open and that not everything that will happen is historically settled. Accordingly, we have a prima facie reason to believe that it is *possible* that the future is open (see the discussion about conceivability above). Even if it should turn out that the future is not open, it *could* have been open. Surely, our best current scientific theories at least *could* have been true.

All arguments in this section can be criticized and since {(1), (2), (3)} is inconsistent, it seems that at least some of the arguments must be unsound. However, together they show that we should take the aporia of omniscience seriously. How should we solve this puzzle? Let us now consider some possible solutions.

3. Possible Solutions

In this section, I will consider nine possible solutions to the aporia of omniscience. Some solutions seem more attractive than others, but no solution is entirely unproblematic. This gives further support to the claim that the aporia of omniscience really is an aporia. According to the first attempt, we should accept dialetheism.

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[6] This argument is not conclusive since there are many different interpretations of modern science. However, it clearly suggests that there is genuine randomness in nature and that the future is open. For more on different interpretations of modern science and some relevant references, see, for example, Birgitte Falkenburg and Friedel Weinert, "Indeterminism and Determinism in Quantum Mechanics," in *Compendium of Quantum Physics*, eds. Daniel Greenberger, Klaus Hentschel and Friedel Weinert (Springer, 2009), 307–311, David Hodgson, "Quantum Physics, Consciousness, and Free Will," in *The Oxford Handbook of Free Will*, ed. Robert Kane (Oxford: Oxford University Press, 2011), 57–83, Wesley C. Salmon, *Causality and Explanation* (Oxford: Oxford University Press, 1998), Ch. 2.

Solution 1: Accept dialetheism

One solution to the problem of omniscience is to accept the idea that there are true contradictions. Some philosophers, so-called dialetheists, believe that there are sentences that are both true and false. If we accept this idea, we might also accept the idea that it is possible to derive a contradiction from {(1), (2), (3)} even though all sentences in this set are true. This might be perfectly acceptable if there are true contradictions.

Nevertheless, this solution is quite problematic. Dialetheism is a very controversial theory and most dialetheists probably agree that not *every* contradiction is genuine (true). Hence, even a dialetheist might think that it is problematic that {(1), (2), (3)} is inconsistent.[7] So, it seems doubtful that this solution should turn out to be the most plausible overall.

Solution 2: Reject (1) because the concept of omniscience is incoherent

According to the second solution, we should reject (1) because the concept of omniscience is incoherent. It only seems to be possible that there is someone who is necessarily omniscient and infallible, but when we analyse the concept closer we can see that it is incoherent. There are other arguments in the literature that suggest that it is problematic to assume that it is possible that there is someone who is omniscient. It has, for example, been suggested that the concept of omniscience is inconsistent with so-called *de re* and *de se* beliefs, with human freedom and with the fact that there is no set of all truths.[8] So, maybe it is reasonable to reject (1) for many different reasons.

However, it certainly seems to be possible that there is someone who is necessarily omniscient and infallible. So, if we can reject (1) but accept some similar proposition instead of claiming that the concept of omniscience is incoherent, this appears to be a prima facie more plausible solution. Accordingly, our next solution might be better.

[7] For more on dialetheism, see, for example, Graham Priest, Francesco Berto, Zach Weber, "Dialetheism," in *The Stanford Encyclopedia of Philosophy*, ed. Edward N. Zalta (2019 Edition), URL = < https://plato.stanford.edu/entries/dialetheism/>.

[8] For more on this, see, for example, Edward R. Wierenga, "Omniscience," in *The Oxford Handbook of Philosophical Theology*, eds. Thomas P. Flint and Michael C. Rea (Oxford and New York: Oxford University Press, 2009), 129–144. It is beyond the scope of this paper to discuss these arguments in detail.

Solution 3: Reject (1) but accept some similar proposition

According to the third solution, we reject (1) not because the concept of omniscience is incoherent, but because we should not assume that it is possible that there is someone who is *necessarily* omniscient and infallible. This is compatible with accepting something similar, namely (1′′).

> (1′′) ◆∃x(Ox∧Ix). It is possible that there is some (individual) x such that x is omniscient and x is infallible.

If we reject (1) and accept (1′′), the main argument does not go through; it breaks down at step (14). If this solution is correct, we can solve the aporia of omniscience and still assume that it is possible that there is someone who is omniscient and infallible. We do not have to claim that the concept of omniscience is incoherent. This suggests that solution 3 is better than solution 2. The concept of omniscience does not seem to be incoherent.

Nevertheless, this solution is not unproblematic, since we still have to reject (1), which is an intuitively attractive proposition. So, let us see if there are any other possible solutions to the puzzle.

Solution 4: Reject (2) because not all beliefs are historically settled

According to the fourth solution, we should reject the idea that it is necessary that all our beliefs are historically settled, that is, we should reject proposition (2). If (2) is not true, then the following sentence is true '◆∃x∃A(BxA∧¬□BxA),' which says that it is possible that there is someone who believes some proposition even though it is not historically settled that she believes this proposition. (2) might be false because at least some of our beliefs might depend on what happens in the future in the sense that what we believe now depends on what will actually happen later on. This idea is strange, but perhaps not so strange that we should reject it. Some propositions about the future certainly seem to be historically open, for example, the judgement that a democrat will win the next election. It is not (now) historically settled whether this proposition is true or not. If a democrat will win the next election, we can say that the proposition that a democrat will win was true (now) once it is settled that the democrat did win. Moreover, if a democrat will not win the next election, we can say that the proposition that a democrat will win was not true (now) once it is settled that the democrat did not win. Right now, there is no fact of the matter whether this proposition is settled true or not; we have to wait to see who is going to win to 'decide' its truth-value. Propositions about our beliefs could behave in a similar way. Perhaps we can believe some propositions even though it is not historically settled that we believe them. If this is true, we can solve the aporia of omniscience. Given that we reject (2), we must also reject (SC), since (SC) entails (2). However, this might be a small price to pay if we are willing to give up the idea that our beliefs are historically settled.[9]

A problem with this solution is that there appear to be important and relevant differences between the proposition that a democrat will win the next election and the proposition that an individual *believes* that a democrat will win the next election. It is reasonable to assume that the first proposition is not historically settled if the future is open, for this proposition tells us something about the future. Nevertheless, the second proposition does not seem to assert anything about what will happen later on. It only says something about what someone *believes* about the future. Our current beliefs (whatever their contents) do not seem to depend on the future. We might have to give up this view since {(1), (2), (3)} is inconsistent, but before we do that, let us see if there are any other solutions to the aporia.

Solution 5: Reject (3) and assume some similar principle

According to the fifth solution, we should reject (3) but assume some similar principle. It is not possible that the future is open in the sense that (3) is true, but it might be possible in some other sense. (3) is not a plausible explication of what it means to say that it is possible that the future is open.

The problem with this solution is that it is not easy to come up with some other reasonable explication. We could perhaps say that the future is open in a possible world w at a time t iff there is some proposition A such that it is historically possible that it will be the case that A and it is historically possible that it will not be the case that A in w at t. In other words, the future is open in w at t iff $\exists A(\Diamond FA \wedge \Diamond\neg FA)$ is true in w at t. Maybe (1) and (2) is consistent with (3′) = $\blacklozenge\exists A(\Diamond FA \wedge \Diamond\neg FA)$ even though {(1), (2), (3)} is inconsistent. $\blacklozenge\exists A(FA \wedge \neg\Box FA)$ entails $\blacklozenge\exists A(\Diamond FA \wedge \Diamond\neg FA)$, but not vice versa. However, this particular suggestion does not solve the problem, for {(1), (2), (3′)} is also inconsistent (the proof of this is left to the reader). So, unless someone can come up with an alternative analysis of

[9] Solution 4 can perhaps be called the Ockhamist solution since it is similar to a solution that Ockham suggested to a similar problem. For more on this, see William of Ockham, *Predestination, God's Foreknowledge, and Future Contingents. Translated with an Introduction, Notes, and Appendices by Marilyn McCord Adams and Norman Kretzmann*, 2nd ed. (Indianapolis: Hackett Publishing, 1983), especially Question II, Article IV. However, it is not obvious how Ockham should be interpreted and the problem he discusses is not exactly the same as the aporia of omniscience.

what it means to say that it is possible that the future is open that is plausible, it seems that we cannot use this solution to the aporia of omniscience.

Solution 6: Reject (3) and assume that it is not possible that the future is open

According to the sixth solution, we should reject (3) and not assume any similar principle. It is not possible that the future is open in the sense that (3) is true, and it is not possible in any other interesting sense either. Since {(1), (2), (3)} is inconsistent, we must probably give up some of our intuitions. So, maybe we should accept the 'fact' that it is not possible that the future is open.

However, this solution is counterintuitive and if we accept it, we must reject the argument from science and the argument from conceivability for (3) (see above). And those arguments appear to be particularly strong. If we accept this solution, it seems that we have to conclude that it is not conceivable that the future is open or that conceivability in this case does not entail possibility. It also appears to be the case that we must deny that our best current science shows that the future is open or else deny that the fact that the future is open entails that (3) is true. Consequently, this solution comes with a very high price.

Solution 7: Reject the definition of infallibility

It is possible to solve the puzzle by rejecting the definition of infallibility (I). If we use the following alternative analysis of this concept, the main argument does not go through any more:

(I′) $\blacksquare\forall x(Ix\leftrightarrow\forall A(BxA\rightarrow A))$. It is necessary that for every (individual) x: x is infallible if and only if everything x believes is true.

According to this definition, it is possible that someone is infallible in a possible world at a moment of time without being infallible in every possible world at every moment in time. Suppose that we use (I′) and not (I) to define the concept of infallibility. Then step (25) in the main argument breaks down. Hence, {(1), (2), (3)} is no longer inconsistent.

Nevertheless, it is doubtful that this solution is plausible, since we can show that some similar sets are inconsistent. Suppose that we replace (1) by (1′) = $\blacklozenge\exists x(\blacksquare Ox\wedge\blacksquare Dx)$ or by (1′′′) = $\blacklozenge\exists x(\blacksquare Ox\wedge\blacksquare Ix)$ and (I) by (I′). Then the following sets are inconsistent: {(1′), (2), (3)} and {(1′′′), (2), (3)} (proofs are left to the reader). So, even though {(1), (2), (3)} is no longer inconsistent if we use definition (I′), at least two other, equally problematic, sets of sentences are inconsistent given definition (I′). A more plausible solution might be to reject the definition of omniscience instead of the definition of infallibility.

Solution 8: Reject the definition of omniscience

According to the eight solution, we should reject the definition of omniscience. Perhaps omniscience does not require knowledge of absolutely *all* truths, perhaps it only requires knowledge of *some* truths. Consider the following alternative analysis of the concept:

> (O′′) ■∀x(Ox↔∀A(□A→KxA)). It is necessary that for every (individual) x: x is omniscient if and only if (iff) for every (proposition) A, if it is historically settled that A (is true) then x knows that A.

If we use this definition of omniscience instead of (O), we cannot use the main argument to prove that {(1), (2), (3)} is inconsistent any longer. At step (16) we would arrive at '□FX→BxFX…' rather than 'FX→BxFX… .' According to this solution, an omniscient being will only have knowledge about truths about the future that are historically settled. Suppose it is not historically settled that a democrat will win the next election. Then an omniscient individual will not know that a democrat will win the next election (nor will this individual know that a democrat will not will). Such a person can still know many propositions about the future that are historically settled, for example, that it will be the case that a democrat will win or that a democrat will not win, that it will be the case that 1 + 1 = 2, that it will be the case that $E = mc^2$, etc. Nevertheless, if it is not historically settled that it will be the case that A, then not even an omniscient individual will know that it will be the case that A. Consequently, if there are truths about the future that are not historically settled, then not even an omniscient individual will have knowledge of such truths.

This solution is attractive in many respects. If it is not settled yet whether or not a democrat will win the next election, how could anyone know that a democrat will (or will not) win. Still, if we accept this solution, we have to assume that an omniscient individual's knowledge of the future is very limited. She (or he) will not know anything about the future that is not historically settled (or, at least, it is not necessary that she (he) has such knowledge). This seems counterintuitive.

Solution 9: Reject the principle of bivalence for future contingents

According to the ninth solution, we should reject the principle of bivalence for future contingents. There are no historically contingent truths about the future. If it is historically possible that a democrat will win and it is historically possible that a democrat will not win, then the sentence 'A democrat will win the next election' is neither true nor false (nor is the sentence 'A democrat will not win the next election' true or false). How does this solve the aporia of omniscience? Well, if there are no truths about the (historically) contingent future, the propositional (or sentential) quantifier in the definition of omniscience (O) does not range over absolutely every proposition (sentence). Suppose that the individual i is omniscient. Then i knows every truth, that is, then $\forall A(A \rightarrow KiA)$ holds. However, since not every sentence about the future is either true or false, we cannot instantiate $\forall A(A \rightarrow KiA)$ with any sentence whatsoever. In particular, we cannot instantiate this sentence with FA, that is, $FA \rightarrow KiFA$ does not follow from $\forall A(A \rightarrow KiA)$. FA may speak about a contingent truth about the future, for example, that a democrat will win the next election. If this is correct, step (16) does not follow from step (15) in the main argument. Hence, the derivation does not go through. Consequently, we can avoid the aporia of omniscience by denying that an omniscient individual has knowledge of the (historically) contingent future. This does not entail that an individual of this kind is less than omniscient, for there are no truths about the contingent future for this individual to know.[10]

A problem with this solution is that we have to assume that the principle of bivalence is false for some sentences. And this principle is intuitively very reasonable and a part of standard (propositional) logic.

4. Conclusion

In this paper, I have discussed a new aporia, the aporia of omniscience. This aporia includes three propositions: (1) It is possible that there is someone who is necessarily omniscient and infallible, (2) It is necessary that all beliefs are historically settled, and (3) It is possible that the future is open. Every sentence in this set is intuitively reasonable and there are prima facie plausible arguments for each of them. However, the whole set {(1), (2), (3)} is inconsistent. Therefore, it seems to be the case that at least one of the propositions in this set must be false. I

[10] The idea that propositions about the contingent future are neither true nor false is old. It has been argued that already Aristotle defended this position (even though there are other interpretations of the Greek philosopher). For more on this, see Aristotle, *Aristotle's "Categories" and "De Interpretatione," Translated with Notes and Glossary by John Lloyd Ackrill* (Oxford: Clarendon/ Oxford Press, 1963). In the 20th century Jan Łukasiewicz suggested that we need a three-valued logic to deal with similar problems (see papers in Storrs McCall (ed.), *Polish Logic 1920–1939* (Oxford: Clarendon Press, 1967)), a view that seems to have been shared by Arthur N. Prior in the 1950's (Arthur N. Prior, "On Three–Valued Logic and Future Contingents," *The Philosophical Quarterly*, Vol. 3 (1953): 317–326). For more on the history of this idea and many relevant references, see, for example, William Lane Craig, *The Problem of Divine Foreknowledge and Future Contingents from Aristotle to Suarez* (Leiden: E. J. Brill, 1988) and Peter Øhrstrøm and Per Frederik Vilhelm Hasle, *Temporal Logic: From Ancient Ideas to Artificial Intelligence* (Dordrecht/Boston/London: Kluwer Academic Publishers, 1995).

have discussed nine different possible solutions to this aporia and I have considered some arguments for and against these solutions. Some solutions seem more promising than others. The solutions that reject (3) are, for example, quite problematic since they seem to contradict our current best science. And the first solution is reasonable only if dialetheism, which is a quite controversial theory, is plausible. However, it is not obvious which solution is the best all things considered and there are arguments against all of them. The fact that it is difficult to tell which solution we should choose reinforces the claim that the aporia of omniscience really is an aporia. No matter how we choose to solve the puzzle, it seems that we have to give up some of our intuitions.[11]

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