

# QUINE AND THE INCOHERENCE OF THE INDISPENSABILITY ARGUMENT

Michael J. SHAFFER

ABSTRACT: It is an under-appreciated fact that Quine's rejection of the analytic/synthetic distinction—when coupled with some other plausible and related views—implies that there are serious difficulties in demarcating empirical theories from pure mathematical theories within the Quinean framework. This is a serious problem because there seems to be a principled difference between the two disciplines that cannot apparently be captured in the orthodox Quinean framework. For the purpose of simplicity let us call this *Quine's problem of demarcation*. In this paper this problem will be articulated and it will be shown that the typical sorts of responses to this problem are all unworkable within the Quinean framework. It will then be shown that the lack of resources to solve this problem within the Quinean framework implies that Quine's version of the indispensability argument cannot get off the ground, for it presupposes the possibility of making such a distinction.

KEYWORDS: mathematics, indispensability, knowledge, holism, confirmation, ontology

## 1. Introduction

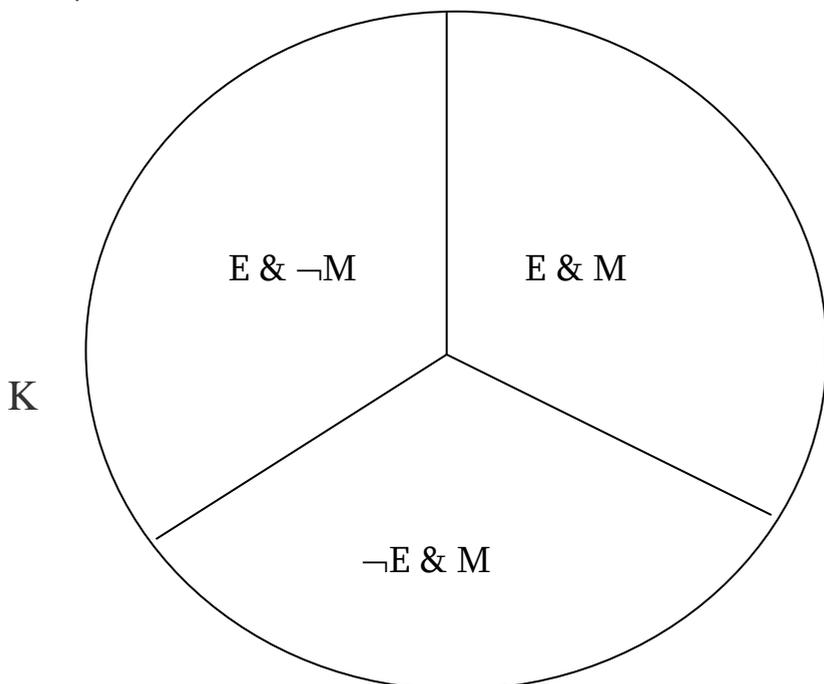
It is an under-appreciated fact that Quine's rejection of the analytic/synthetic distinction—when coupled with some other plausible and related views—implies that there are serious difficulties in demarcating empirical theories from pure mathematical theories within the Quinean framework.<sup>1</sup> This is a serious problem because there seems to be a principled difference between the two disciplines that cannot apparently be captured in the orthodox Quinean framework. For the purpose of simplicity let us call this *Quine's problem of demarcation*. In this paper this problem will be articulated and it will be shown that the typical sorts of responses to this problem are all unworkable within the Quinean framework. It will then be shown that the lack of resources to solve this problem within the Quinean framework undermines Quine's version of the indispensability argument, for it presupposes the possibility of such a distinction.

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<sup>1</sup> See W.V.O. Quine "Two Dogmas of Empiricism," *Philosophical Review* 60 (1951): 20-43.

## 2. Quine and the Problem of Demarcating Science and Mathematics

Quine is duly famous for his critique of the analytic/synthetic distinction in his 1951 article “Two Dogmas of Empiricism,” despite persistent disagreement about the significance of this work. Nevertheless, given Quine’s criticism of the analytic/synthetic distinction, Quine (and those who follow in his naturalistic footsteps) regard *all* statements as being empirical in character in some important sense. Thus, every statement is supposed to be subject to revision in light of empirical evidence. Of course, this is not news to us today, but what this implies about teasing apart pure mathematics from empirical theory has not been properly appreciated. What it immediately and most obviously implies is that Quineans cannot discriminate pure mathematics from empirical science by asserting that the propositions that make up mathematical theories are analytic, whereas the propositions that make up empirical theories are synthetic. This is troubling because in practice there appears to be a quite sharp distinction between the practices of mathematics and the empirical sciences. Presumably we would then like to be able to partition the complete body of known statements  $K$  into  $M$ —the mathematical statements—and  $E$ —the empirical statements, such that  $E \subset K$  and  $M \subset K$ . But, in order to accomplish this task, a plausible criterion  $C$  that grounds the distinction between the elements of  $E$  and  $M$  must exist. This figure captures the necessary distinction:



Here, according to the Quinean view, we are *only* to adopt ontological commitment to those claims that lay within the E regions of K. What we then need, in the spirit of Quine's "no entity without identity" dictum, is a criterion of identity that could demarcate mathematical statements from empirical statements (and thereby also a related criterion to demarcate the statements of pure mathematics from those of applied mathematics). Without such an identity criterion, Quineans would violate their own ontological scruples. Quine's criticism of the analytic/synthetic distinction immediately shows that Quineans cannot use analyticity as the criterion for demarcating mathematical statements from empirical statements in K and so we must look elsewhere if we are to solve the Quinean problem of demarcation

This inability to discriminate pure mathematics from empirical science on the basis of the analytic/synthetic distinction in the Quinean framework is then exacerbated further when it is also recognized that traditional methodological accounts of pure mathematics are wildly unrealistic and depend on the viability of a workable notion of a priority. Those accounts typically treat the methodology of pure mathematics as the development of necessarily true axiomatic systems where theorems are proved on the basis of the axioms by the use of a priori methods, but this is simply not true of the actual practice of mathematics as numerous philosophers of mathematics have now come to realize. Lakatos in particular is largely responsible for this recognition.<sup>2</sup>

More importantly, Quine's rejection of the analytic/synthetic distinction also simultaneously eliminates the viability of using either the a priori/a posteriori distinction or the necessary/contingent distinction as the basis for demarcating empirical science from pure mathematics because, at least for Quineans, as a result of the collapse of the analytic/synthetic distinction, there is no a priori knowledge and there are no necessarily true propositions. Quine held that the class of analytic truths is just the class of a priori knowable truths and the class of a priori knowable truth is just the class of necessary truths. Similarly, the class of synthetic truths is just the class of a posteriori knowable truths and the class of a posteriori knowable truths is just the class of contingent truths. But according to Quine there

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<sup>2</sup> See Imre Lakatos, "Proofs and Refutations," *British Journal for the Philosophy of Science* 14 (1963-4): 1-25, 120-139, 221-243, 296, 342, Imre Lakatos, *Proofs and Refutations* (Cambridge, MA: Cambridge University Press, 1976) and Imre Lakatos, "A Renaissance of Empiricism in the Recent Philosophy of Mathematics," *British Journal for the Philosophy of Science* 27 (1976): 201-223.

are no analytic truths and so there are no truths that are knowable a priori and there are no necessary truths. So, as we saw earlier Quineans cannot use the analytic/synthetic distinction to do the work of C and for the same basic reason they can use neither the a priori/a posteriori distinction nor the necessary/contingent distinction to do the work of C. Moreover, they cannot use any criterion that employs any of these conceptual distinctions, for there are no such distinctions according to Quine.<sup>3</sup>

Yet more troubling still is the recognition that Quine's rejection of the analytic/synthetic distinction also implies a radical sort of confirmational holism, whereby the whole of our system of beliefs is the proper unit of confirmation relative to observational data.<sup>4</sup> In other words the whole body of our beliefs is at issue when it comes to the issue of confirmation. This means that when we consider the acceptability of our beliefs it must be done in a global manner and when our system of beliefs conflicts with observational data we are then confronted with the infamous Quine/Duhem thesis.<sup>5</sup> This is the assertion that

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<sup>3</sup> Quine makes an alternative and more mature attempt to specify C later on in W. V. O. Quine, *From Stimulus to Science* (Cambridge, MA: Harvard University Press, 1995), 52-57. There he argues that the distinction between empirical statements and those of pure mathematics can perhaps be established as follows. Empirical statements and sets of empirical statements imply observation categoricals, whereas the statements of pure mathematics individually and jointly do not. This suggestion is variously problematic as follows. First, this approach fails to specify a defining feature of specifically mathematical statements, as opposed to other non-empirical statements. Moreover, it makes the statements of pure mathematics meaningless and devoid of truth values given his adherence to the view that all semantic content is ultimately grounded empirically. Finally, semantic content is typically generated only by sets of statements that must involve at least some empirical claims, but in Quine's system there is no way in practice to tease apart which statements in a testable set with semantic content are specifically conferring the empirical content on that set. This is due to his subscription to a form of semantic holism. We cannot in most cases selectively and sequentially delete statements and then check to see if semantic content remains and this is simply because critical semantic mass is not a property of single sentences (*From Stimulus to Science*, 48-49). Elsewhere, he famously tells us that "It is misleading to speak of the empirical content of an individual statement ("Two Dogmas," 43)," and also that "the unit of empirical significance is the whole of science ("Two Dogmas," 42)." In fact, Quine admits all of these charges (*From Stimulus to Science*, 55-57) and so even he acknowledges that this approach is problematic.

<sup>4</sup> In Quine's later work this more extreme view is relaxed and Quine holds that "large" chunks of our systems of belief are the units of confirmation. See W.V.O. Quine, "Five Milestones of Empiricism," (1975) reprinted in *Theories and Things* (Cambridge: Harvard University Press, 1981).

<sup>5</sup> See W.V.O. Quine, *Philosophy of Logic* (Englewood, NJ: Prentice-Hall, 1970) and W.V.O. Quine and Joseph Ullian, *The Web of Belief* (New York: Random House, 1970).

when our holistic system of beliefs is faced with empirical falsification, we must give up something to restore consistency, but that there is nothing *in particular* we must give up. We can give up the observation statement itself or one or more theoretical beliefs that give rise to the contradiction. As a result, when our system of beliefs is faced with falsification we must give *something* up, but we can typically restore consistency in a number of ways by adjusting our beliefs. This view then also undermines the principled possibility of using the concept of revisability to make the distinction between the statements of mathematics and those of science.

What is of great interest is that in discussing the role of mathematics and its ontology in the context of its application in empirical theory, Quine—along with Putnam—subscribed to the infamous indispensability arguments.<sup>6</sup> Such indispensability arguments take the following generic form:

P1: We ought to have ontological commitments to all and only entities that are indispensable to our best scientific theories.

P2: Mathematical entities are indispensable to our best scientific theories.

Therefore, we ought to have ontological commitment to mathematical entities.

These arguments essentially conclude that our ontological commitments to the existence of mathematical entities should be on a par with our ontological commitments to the theoretical entities appealed to in empirical theories because indispensable mathematical propositions employed in such theorizing accrue confirmation holistically when the empirical propositions in question are confirmed. That is to say, all of the propositions used in some given empirical endeavor—including those that are a part of mathematics—accrue confirmation *jointly* because no proposition can be confirmed in isolation and mathematics is in some crucially important sense indispensable to the conduct of the work of the empirical sciences.

But this also means that the difference between the statements of pure mathematics and the statements of the empirical sciences also cannot be grounded

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<sup>6</sup> For the origin of the indispensability arguments see W.V.O. Quine, “On What There Is,” *Review of Metaphysics* 2 (1948): 21-38, “Two Dogmas,” *Word and Object* (New York, MIT Press, 1960), “Carnap and Logical Truth,” *Synthese* 12 (1960): 350-374, *Ontological Relativity and Other Essays* (New York: Columbia University Press, 1969), *Theories and Things* (Cambridge: Harvard University Press, 1981), Hilary Putnam, *Philosophy of Logic* (New York: Allen and Unwin, 1972), *Mathematics, Matter and Method*, 2nd ed. (Cambridge, MA: Cambridge University Press, 1979). For discussion of these arguments see Mark Colyvan, *The Indispensability of Mathematics* (Oxford: Oxford University Press, 2003) and Penelope Maddy, *Realism in Mathematics* (Oxford: Clarendon Press, 1990).

in the differing ontological attitudes towards the referents of mathematical statements and the referents of empirical statements, for what we should take to exist is just what our best total theory of the world indicates as existing.<sup>7</sup> So given the Quinean stance on ontology the criterion C cannot be difference in ontological attitude, for there are no such differences. But, commitment to the indispensability arguments strongly indicates that Quine and those who follow his lead must identify some principled manner by which mathematical propositions can be distinguished from other propositions. Otherwise, the indispensability arguments verge on being nonsensical, for their very formulation depends on such a distinction. In the indispensability argument both P1 and P2 *assume* such a distinction. In order to make the indispensability argument work we must already have in hand a viable criterion C by which we can distinguish statements involving mathematical entities from statements of empirical science. Otherwise we would have no way to establish P2, i.e. that some *specifically* mathematical entities are indispensable to any scientific theory. This is because we would have no way to determine which statements concern mathematical entities and it might turn out that unbeknownst to us there *are no mathematical entities involved in science at all*. If that were true, then the indispensability argument would just be pointless. Similarly, absent some criterion to ground the principled difference between mathematical and scientific statements we would have no way to determine that we ought to be ontologically committed to. This is because we could not follow P1—the claim that the only entities that we should be committed to are those that are indispensable only to our scientific theories—in practice if we do not know what count as specifically scientific claims and what count as statements of pure mathematics.

What all of this ultimately appears to imply about discriminating pure mathematics from empirical science within the Quinean framework should now be apparent and it is deeply troubling. Namely, there is no obvious way to ground the clearly real and important distinction between the empirical sciences and pure mathematics in light of orthodox Quinean principles. Quine explicitly recognizes in his 1951 article that the statements of pure mathematics are as revisable as those of any empirical science, but does not appear to see how deeply that this threatens the *very distinction* between the two types of statements and between the two disciplines. What he does not recognize is that his views, when taken together, threaten the identity conditions for mathematical and empirical statements. Quine himself seems only to have recognized that at the superficial level the difference between the mathematical elements of our belief systems and the more properly

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<sup>7</sup> See Quine, “On What There Is.”

empirical elements is one of degree of entrenchment in our belief system. More specifically, typically then the propositions of mathematics are supposed to be more deeply entrenched than the propositions of empirical theories. But, that is just to say that when our belief system faces falsification we are (typically) *psychologically* less inclined to give up the propositions of pure mathematics than we are to give up observation statements or more properly empirical statements of the empirical sciences. But, this is nothing more than a difference in psychological attitudes towards those types of statements which (1) is itself totally ungrounded on a theoretical level—although it does serve to explain the “felt” necessity of mathematics, and (2) which may vary across individuals. Nevertheless, this simply and directly implies that there is no obvious, principled and theoretically grounded criterion C that can be used to make distinction between the statements of pure mathematics and the statements of the empirical sciences within the Quinean corpus. So, for Quineans it appears to be the case that there really is no identifiable principled difference between science and pure mathematics. But, this conclusion is wildly implausible given the practice of both mathematics and the empirical sciences and it renders the indispensability argument incoherent. Moreover, if Quineans simply fall back to the view that conformational holism is ultimately the source of support for all statements in the web of belief—including those of mathematics—and thus fixes ontological commitment, it would simply be a concession that the indispensability argument is simply irrelevant because that would require conceding P1. Doing so would also presumably entail the kind of indiscriminate and comprehensive Platonism that is at odds with the conservative nature of Quine’s attitude towards the existence of mathematical entities in particular and abstract objects in general as forcefully argued for in his 1960a and which is clearly precluded by P1. Quine wants us to commit *only* to the existence of those mathematical entities that are in fact indispensable to science and not indiscriminately to the existence of all mathematical entities. So, especially given the “no entity without identity” principle, Quineans need to acknowledge the problem and to provide a principled manner in which pure mathematics can be distinguished from empirical science that retains all—or at least most—of the basic Quinean views and which allows us to make sense of the indispensability arguments. However, it is not at all clear how this might be accomplished while retaining the core of Quine’s argument against the viability of the analytic/synthetic distinction. So, Quine’s views may simply be inconsistent when it comes to this particular issue.