AN AXIOM LINKING NECESSITY AND OBLIGATION PROVIDED BY PRIOR AND ITS ANALYSIS UNDER CARNAP'S METHOD

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ABSTRACT: Although written long before, in 2012 a work by Prior presenting a system that was able to demonstrate Hintikka's theorem was published. Maybe one of the most relevant elements of that system is an axiom that clearly relates necessity, and hence modal logic, to obligation, and hence deontic logic. This paper analyzes that axiom based upon Carnap's method of extension and intension in order to show that it should be accepted. Thus, the paper is intended to give further evidence supporting not only the aforementioned axiom, but also Prior's system in general and, accordingly, Hintikka's theorem.

KEYWORDS: Rudolf Carnap, L-concepts, necessity, obligation, Arthur N. Prior

Introduction

There is no doubt that the theorem known as 'Hintikka's theorem' is, at least, disturbing. The reason for this is that the theorem provides that, if something is not possible, it cannot be allowed, an idea that, in principle, seems to be very rare. However, Prior thought about a system that proved it. Although probably written some decades before,[1] the paper[2] was not published until 2012, when it was, in addition, reviewed in detail in another paper of the same issue in the same journal.[3] Besides, the system has also been addressed in another work that has tried to show that its real potential is actually great and to account for why, despite that

[1] See Peter Øhrstrøm, Jörg Zeller, and Ulrik Sandborg-Petersen, "Prior's Defence of Hintikka's Theorem. A Discussion of Prior's 'The Logic of Obligation and the Obligations of the Logician'," *Synthese* 188, 3 (2012): 449-454.

[2] Arthur N. Prior, "The Logic of Obligation and the Obligations of the Logician," *Synthese* 188, 3 (2012): 423-448.

[3] Øhrstrøm et al., "Prior's Defence of Hintikka's Theorem," 449-454.

potential, people tend to reject its main consequence: Hintikka's theorem.[4] The explanation about that tendency to rejection in this last work was based upon the mental models theory,[5] a psychological theory basically stating that human reasoning is not logical and that essentially works taking semantic (in the linguistic sense of this word) possibilities into account. Nevertheless, that explanation concluded with a proposal to make people really understand Hintikka's theorem, which consisted of offering clear definitions for concepts such as 'permitted' or 'forbidden.' In this way, the chief idea was that concepts such as those ones were interpreted as referring to an impossibility to do a particular action in practice, that is, to a physical impossibility.

Nonetheless, the aim of this paper is only to give further support to the theorem and the demonstration of it that can be derived in Prior's system. In particular, it tries to reveal that, from other frameworks or methods, that theorem and that demonstration could also appear to be correct and acceptable. This will be done by means of the analysis of a concrete axiom in the system given by Prior and resorting to a particular approach of contemporary philosophy. The axiom is the one that can be deemed as the most relevant axiom in Prior's system, since, while obviously all of the elements in that system are important, it is the axiom that explicitly links the concept of necessity (and therefore a machinery such as the one that modal logics can provide today) to obligation (and therefore to a number of logical resources such as the ones that deontic logic can give nowadays). On the other hand, the approach is the well-known method of extension and intension proposed by Carnap.[6]

Of course, it is clear that a similar theoretical task could be done by considering other axioms (or even other elements) in Prior's system and other methods of analysis of meanings. However, the study that will be carried out here

[4] Miguel López-Astorga, "What Is Possible and What Is permitted: Hintikka and Prior," *Analele Universitatii din Craiova, Seria Filosofie,* 39, 1 (2017): 57-66.
[5] See, e.g., Sangeet Khemlani, Thomas Hinterecker, and Philip N. Johnson-Laird, "The Provenance of Modal Inference," in *Proceedings of the 39th Annual Conference of the Cognitive Science Society*, eds. Glenn Gunzelmann, Andrew Howes, Thora Tenbrink, and Eddy J. Davelaar (Austin: Cognitive Science Society, 2017), 259-264; Ana Cristina Quelhas and Philip N. Johnson-Laird, "The Modulation of Disjunctive Assertions," *The Quarterly Journal of Experimental Psychology* 70, 4 (2017): 703-717; Ana Cristina Quelhas, Célia Rasga, and Philip N. Johnson-Laird, "A Priori True and False Conditionals," *Cognitive Science* 41, 55 (2017): 1003-1030.
[6] Rudolf Carnap, *Meaning and Necessity: A Study in Semantics and Modal Logic* (Chicago: The University of Chicago Press, 1947).

by using the mentioned axiom and framework can be illustrative enough to lead to think that more developments in this way, although they can always be appropriate and interesting, would be trivial and superfluous. In any case, to do what has been said, firstly the axiom will be described and, then, given that, as shown below, it has a conditional formal structure, two options will be taken into account. One of them will be the hypothetic scenario in which its antecedent is true, and the other one will be the possible alternative in which it is false. Evidently, each of these two options will be reviewed paying attention to the consequences that can be drawn from the theses and definitions included in Carnap's method and which can be related to what is provided by the axiom, the goal being, as stated, to show that that axiom is compatible with and can be assumed under a method such as that of Carnap. But the next section begins with the first action to do, which is, as also indicated, to explain what the axiom is exactly.

The Link between Necessity and Obligation in the System Proposed by Prior[7]

The particular axiom that will be addressed here is maybe the most important element in Prior's system, since it is, as pointed out, the element that relates modal logic to deontic logic in that system, and, accordingly, the element that, after all, allows relating what is possible (or impossible) to what is permitted (or unpermitted), and, in this way, the demonstration of Hintikka's theorem (which is not reproduced here again because, obviously, it is to be found in texts such as some of the ones that have been cited). It is as follows:

$$[I]\ \Box(p \to q) \to (Op \to Oq)$$

(Although with a different symbol for the conditional, [I] is axiom (3) in the text by Øhrstrøm et al.[8]).

The system is based on classical propositional calculus and hence '→' in [I] stands for the material implication in that calculus, or, if preferred, in frameworks more or less akin to that presented by Deaño[9]. Nevertheless, perhaps the other symbols are more relevant for the aims of this paper. '□' is, as usual in modal logic, the operator of necessity, and, as also customary in that logic, it can be defined by the concept of possibility, whose symbol is '◊':

[7] Prior, "The Logic of Obligation," 423-448.

[8] Øhrstrøm et al., "Prior's Defence of Hintikka's Theorem," 449-454.

[9] Alfredo Deaño, *Introducción a la lógica formal* (Madrid: Alianza Editorial, 1999).

[II] $\Box(x) =_{df} \neg\Diamond\neg(x)$

(Where, obviously, '¬' represents negation and the 'x' between brackets to any well-formed formula in classical propositional logic).

And, as it is well known, possibility can also be defined by virtue of necessity in modal logic:

[III] $\Diamond(x) =_{df} \neg\Box\neg(x)$

As far as 'O' is concerned, clearly, it is the symbol of obligation in deontic logic, and, in this last logic, 'P', that is, the symbol standing for permission, habitually defines it:

[IV] $O(x) =_{df} \neg P\neg(x)$

But, as in the previous case, 'P' can be defined using 'O' too:

[V] $P(x) =_{df} \neg O\neg(x)$

Thus, it is absolutely clear the sense of [I]. It provides that the fact that a conditional is necessary implies that, if its antecedent is obligatory, then its consequent is obligatory as well. A formula such as that is not really hard to accept, and that can be seen by means of a method such as the one of Carnap.

L-truth and the Case in Which the Antecedent is L-true

A very important aspect of Carnap's method is that it includes L-concepts. Those are concepts that, in a similar way as Kantian analytical judgments, are correct a priori and just by virtue of their meanings in the particular language that is being used. So, given a number of 'state-descriptions' (which is the expression to which Carnap resorts to indicate something similar to what the possible worlds are in modal logic), a L-concept is a concept that is correct in all of the state-descriptions. In this way, it can be said, for example, that (x) is L-true if and only if (x) is true in all of the state-descriptions that can be thought (see, e.g., Definition 2.2 in the text by Carnap[10]), and this leads one to note that what $\Box(x)$ actually provides is that (x) is L-true (see, e.g., Convention 39-3 in the text by Carnap[11]).

Nonetheless, what is interesting now is that just a few notions such as these ones coming from the method of extension and intension can be sufficient to show that [I] should be accepted. Two possibilities can be thought in this regard: that its

[10] Carnap, *Meaning and Necessity*.
[11] Carnap, *Meaning and Necessity*.

antecedent, $\Box(p \rightarrow q)$, is true and that it is false. The first case will be dealt with in this section and the second one in the next section.

If a formula such as $\Box(p \rightarrow q)$ is true, that means that $p \rightarrow q$ is true in all of the state-descriptions and that hence a state-description in which it is false cannot be thought ($p \rightarrow q$ is L-true). However, in classical logic, implication, as said, is material, and, as it is well known, that in turn means that,

[VI] $(x) \rightarrow (y) =_{df} \neg[(x) \wedge \neg(y)]$

(Where '$\wedge$' stands for conjunction).

Thus, if the antecedent of [I], $\Box(p \rightarrow q)$, is true, by [VI], its consequent, $Op \rightarrow Oq$, also has to be true. In fact, it has to be true in exactly the same cases as the antecedent. Accordingly, since the antecedent is L-true, the consequent needs to be so too, and, therefore, $Op \rightarrow Oq$ has to be true in all of the state-descriptions as well. And this leads to another important concept in Carnap's framework, which is the concept of L-equivalence: two formulae are L-equivalent when the state-descriptions in which they are true are exactly the same (see, e.g., Result 2-6 in the text by Carnap[12]). So, it can be said that this formula is L-true:

[VII] $\Box(p \rightarrow q) \leftrightarrow (Op \rightarrow Oq)$

(Where '$\leftrightarrow$' is the symbol of the biconditional relationship).

But, following classical propositional calculus, this formula can also be drawn from [VII] and hence is L-true too:

[VIII] $(Op \rightarrow Oq) \rightarrow \Box(p \rightarrow q)$

and, by [IV], this is another L-true formula:

[IX] $(\neg P\neg p \rightarrow \neg P\neg q) \rightarrow \Box(p \rightarrow q)$

And, by propositional calculus, the same can be claimed for this sentence:

[X] $(P\neg q \rightarrow P\neg p) \rightarrow \Box(p \rightarrow q)$

Even, resorting to an equivalence that is also used in Prior's system to derive Hintikka's theorem (the equivalence, by [II], between $\Box(p \rightarrow q)$ and $\neg\Diamond(p \wedge \neg q)$; see Theorem (4) in the text by Øhrstrøm et al.[13]), it can be stated that the following is a L-true formula as well:

[XI] $(P\neg q \rightarrow P\neg p) \rightarrow \neg\Diamond(p \wedge \neg q)$

[12] Carnap, *Meaning and Necessity*.

[13] Øhrstrøm et al., "Prior's Defence of Hintikka's Theorem," 449-454.

Of course, more combinations are possible based upon the definitions indicated above and classical propositional calculus. However, those that have been pointed out can be enough to show the point of this section. An example with thematic content of a case in which $\square(p \rightarrow q)$ is true, and hence L-true, has to be an example providing a meaning relationship between p and q. In this way, a simple material conditional would not be enough. Maybe it should be a conditional fulfilling criteria such as the one of Chrysippus of Soli[14] or, more recently, the one of the strict implication proposed by Lewis.[15] As it can be noted in his book, Carnap[16] does not seem to avoid discussions in this direction. Nevertheless, perhaps what is important now is to highlight that, if $\square(p \rightarrow q)$ is correct, as said, p and q cannot have any content. Their content has to be such that the combination p and $\neg q$ is impossible (in any state-description that can be thought).

Undoubtedly, different examples with thematic content of $\square(p \rightarrow q)$ being true can be raised. Nonetheless, what is truly interesting here is that any of those examples appears to make sense and to be absolutely coherent not only with [I], but also with formulae [VII] to [XI]. One of those examples can be sufficient to see that. Given a sentence such as this one:

[XII] If I drink rum, then I drink alcohol.

Clearly, it is not possible that the antecedent is true and the consequent is false, as drinking rum necessarily implies drinking alcohol. And these contents for p and q do not seem to cause great difficulties to a formula such as [I], since what this last formula would provide would be that,

[I] The fact that it is necessary that, if I drink rum, then I drink alcohol implies that, if it were obligatory to drink rum, then it would be obligatory to drink alcohol.

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[14] E.g., Jonathan Barnes, Susanne Bobzien, and Mario Mignucci, "Logic," in *The Cambridge History of Hellenistic Philosophy*, eds. Keimpe Algra, Jonathan Barnes, Jaap Mansfeld, and Malcolm Schofield (Cambridge: Cambridge University Press, 2008), 77-225; William Kneale and Martha Kneale, *The Development of Logic* (Oxford: Clarendon, 1962); Robert R. O'Toole and Raymond E. Jennings, "The Megarians and the Stoics," in *Handbook of the History of Logic, Volume 1. Greek, Indian and Arabic Logic*, eds. Dov M. Gabbay and John Woods (Amsterdam: Elsevier, 2004), 397-522.
[15] Clarence Irving Lewis, *A Survey of Symbolic logic* (Berkeley: University of California Press, 1918).
[16] Carnap, *Meaning and Necessity*.

Obviously, it is very hard to think about a world in which drinking rum is mandatory. However, the example appears to be totally coherent and provides an idea that is unquestionably correct: if [XII] is L-true, or, if preferred, if □[XII] is so, then, as indicated, in the hypothetical case in which its antecedent were obligatory, its consequent would be obligatory too. But something similar can be said with regard to [VII]:

> [VII] The fact that it is necessary that, if I drink rum, then I drink alcohol is equivalent (or L-equivalent) to the fact that, if it were obligatory to drink rum, then it would be obligatory to drink alcohol.

Indeed, it is also difficult to imagine a state-description in which, [XII] being true (and, as pointed out, what this example of [VII] clearly states is that it is so in all the state-descriptions), it is not true, at the same time, that the obligation to drink rum would imply the obligation to drink alcohol, and vice versa. In this way, the case with [VIII] would not be very different:

> [VIII] If it is true that, if it is obligatory to drink rum, then it is obligatory to drink alcohol, then it is also necessarily true that, if rum is drunk, then alcohol is drunk.

As claimed for [VII], it would not be an easy task to think about a circumstance in which this instance of [VIII] did not hold, which makes the case of [IX] obvious as well:

> [IX] If it is true that, if it is not permitted not to drink rum, then it is not permitted not to drink alcohol, then it is also necessarily true that, if rum is drunk, then alcohol is drunk.

Maybe any comment on this last example would be trivial, since it is clear that its meaning and sense are not very different from those of the example given for [VIII]. And exactly the same can be stated in connection to [X], whose instance would be:

> [X] If it is true that, if it is permitted not to drink alcohol, then it is permitted not to drink rum, then it is also necessarily true that, if rum is drunk, then alcohol is drunk.

And, finally, the application of the content of [XII] to [XI] also leads to a situation so similar to the previous ones as to make any explanation about it superfluous:

> [XI] If it is true that, if it is permitted not to drink alcohol, then it is permitted not to drink rum, then it is also necessarily true that it is not possible to drink rum

and not to drink alcohol.

So, beyond the mentioned difficulty that to think about a hypothetical situation in which drinking alcohol is mandatory can raise, the examples with thematic content above allow checking that Carnap's semantic method to analyze meanings not only enables to accept the key axiom in Prior's system (and hence, as that fact seems to imply, his demonstration of Hintikka's theorem), but also to consider it to be absolutely suitable. As shown below, this does not greatly change if it is supposed that the antecedent of [I], $\Box(p \rightarrow q)$, is false.

L-truth and the Case in Which the Antecedent is L-false

But, even in the case that $\Box(p \rightarrow q)$ were L-false, [I] would keep being L-true. And the concept used now is 'L-false' because, according to Carnap,[17] a formula is L-false if its negation is L-true, and it is obvious that, if $p \rightarrow q$ is not L-true, $\neg\Box(p \rightarrow q)$ is L-true, and hence $\Box(p \rightarrow q)$ is L-false.[18]

Certainly, as it is well known, in classical propositional calculus, in a consistent way with [VI], $(x) \rightarrow (y)$ is always true when (x) is not. So, if $\Box(p \rightarrow q)$ were untrue, [I] would be, in any state-description, true, no matter what the truth-value of $Op \rightarrow Oq$ were. Accordingly, the problem could be only in the cases [VIII] to [XI], in which $\Box(p \rightarrow q)$ –or, in [XI], the L-equivalent formula $\neg\Diamond(p \wedge \neg q)$- is the consequent and, therefore, one might think, precisely by [VI], that the possibility exists that the formula in its entirety is false. It would be sufficient that the antecedent were true, since, thus, the antecedent would be true and the consequent would be untrue. However, situations such as this last one would not be really possible for at least two reasons.

Firstly, to modify the order of the clauses in a formula such as [I], it has to be transformed into [VII], and this is only possible if both the antecedent and the consequent are true in the same state-descriptions. So, if, for example, in [VIII], $Op \rightarrow Oq$ were true and $\Box(p \rightarrow q)$ were false, that would mean that they are not true in the same state-descriptions, that they are not equivalent (or L-equivalent), and that, therefore, neither [I] could be transformed into [VII] nor [VIII] could be derived from [VII]. And, of course, arguments very akin to these ones apply to formulae [IX] to [XI]. Obviously, the only possibility in which $\Box(p \rightarrow q)$ can be false and equivalent to $Op \rightarrow Oq$ is the case in which $Op \rightarrow Oq$ is false in the same

[17] Carnap, *Meaning and Necessity*, e.g., Definition 2-3-a.

[18] See also, e.g., Carnap, *Meaning and Necessity*, Convention 39-3.

state-descriptions as $\Box(p \rightarrow q)$, that is, in all of the state-descriptions. In this case, antecedent and consequent could change their places in the formula, but, given that, as said, the implication is material in classical propositional calculus, formulae [I] and [VII] to [XI] would continue to be true in all of the possible state-descriptions.

Secondly, if $\Box(p \rightarrow q)$ were L-false, then there would not be semantic relation (whether in the sense indicated by Chrysippus or the one indicated by Lewis or any other) between p and q. In this way, if $p \rightarrow q$ can be false, that is only because the antecedent and the consequent can be linked randomly. And that is what happens in sentences such as this one:

[XIII] If I wear hat, then I wear black shoes.

Clearly, the antecedent and the consequent are not semantically related in [XIII]. Hence they are not so either in [XIV]:

[XIV] If it is obligatory to wear hat, then it is obligatory to wear black shoes.

It is evident that neither [XIII] nor [XIV] can be L-true and that, accordingly, there can be state-descriptions in which one of them is true and the other one is false. Therefore, again, it is not possible to speak about equivalence (or L-equivalence) in the first place, it is not possible to transform [I] into [VII], it is not possible to draw formulae such as [VIII] to [XI] from [VII], and it is not possible that [I] leads to formulae with a true antecedent and a false consequent.

Conclusions

So, this paper can be deemed as one more piece of evidence showing that theorems such as the one of Hintikka may not be absurd and that Prior's system makes sense and has an interesting potential to be used. From a framework, in principle, different from the one of Prior,[19] that of the method of extension and intension provided by Carnap, which, while it takes modality into account, does not consider deontic logic, his axiom seems to be admissible with complete justification. And this, regardless of the fact that it is also, as said, further support for the theorem raised by Hintikka, has two clear consequences.

On the one hand, it appears that Prior's system deserves to continue to be developed. Indeed, it seems to be correct from different perspectives, for example,

[19] Prior, "The Logic of Obligation," 423-448.

from the one adopted by López-Astorga[20] and the one based upon Carnap's semantic method assumed here. Therefore, one might think that maybe, by working under its approach, interesting and surprising conclusions of great relevance in logic, philosophy, and science could be achieved.

On the other hand, this paper also appears to show that a lot of work remains to be done in fields such as linguistics and philosophy of language. As indicated above, the text by López-Astorga[21] tries to clarify what certain words involved in the theorem presented by Hintikka and the system raised by Prior actually mean. That is the case of, for example, 'permitted', which is linked in López-Astorga's paper to senses such as impossibility to do an action from an ontological point of view. Thus, it can be thought that the problems of approaches such as those of Hintikka and Prior are that they refer to several different levels, including physics, metaphysics, ethics, and linguistics, and that perhaps only interdisciplinary studies paying attention to most of those levels can truly reveal all the richness that those approaches have and the real meanings of the words, concepts, and operators used in them, which do not denote exactly the same in all of such levels. In any case, maybe another fact that confirms that the difficulties with these issues are related to the need for a clarification of what certain elements really mean in systems such as the one of Prior is that the method used in this paper is a method to study in-depth and recover the meanings of expressions. So, it is possible that, beyond the method one follows, this is the chief task to do in the near future.[22]

[20] López-Astorga, "What Is Possible and What Is Permitted," 57-66.

[21] López-Astorga, "What Is Possible and What Is Permitted," 57-66.

[22] This paper is a result of the Project CONICYT/FONDECYT/REGULAR/FOLIO № 1180013, "Recuperación de las formas lógicas de los enunciados a partir de un análisis de las posibilidades semánticas a las que hacen referencia", supported by FONDECYT (National Fund for Scientific and Technological Development), Government of Chile.