THE LOGICAL LIMITS OF SCIENTIFIC KNOWLEDGE: HISTORICAL AND INTEGRATIVE PERSPECTIVES

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ABSTRACT: This work investigates some of the most important logical limits of scientific knowledge. We argue that scientific knowledge is based on different logical forms and paradigms. The logical forms, which represent the rational structure of scientific knowledge, show their limits through logical antinomies. The paradigms, which represent the scientific points of view on the world, show their limits through the theoretical anomalies. When these limits arise in science and when scientists become fully and deeply aware of them, they can determine logical or paradigmatic revolutions. These are different in their respective courses, although the logical forms and the paradigms are parts of the same type of knowledge. In the end, science can avoid or can integrate its different limits. In fact, the limits of science can become new opportunities for its growth and development.

KEYWORDS: limits, scientific knowledge, paradigm, logical form, scientific revolution

1. Introduction

In this work, we will investigate the limits of scientific knowledge and in particular its logical limits. We believe, in fact, that scientific knowledge can be described as being composed of a basilar logical form and of one or few paradigms (in the classical, kuhnian, sense of this term). While the logical form refers to the elaboration of knowledge (rational style and structure), the paradigm refers instead to the representation of the same knowledge (point of view on the world, theories and methodologies). Scientific knowledge is limited in both its elaborative and representative capacities. In other words, it shows limits both in its logical forms (logical limits) and in its paradigms (paradigmatic limits). We understand the concept of 'limit' in terms of a border, or a barrier that delimits the extension of a specific field of knowledge.

A large part of our study consists in discussions of the most important and most recent philosophical works on its topic. We will discuss in particular the limits of knowledge listed and described by the Princeton Group (Piet Hut, David

P. Ruelle, and Joseph F. Traub) and the limits of thought listed and described by Graham Priest within the new paraconsistent logic. We believe that the first type of these limits can be interpreted as elaboration limits of scientific knowledge, and so they can be described as logical limits. Moreover, we believe that the second type of these limits can be interpreted as limits of representation of scientific knowledge, and so they can be intended as paradigmatic limits. From our point of view, however, logical and paradigmatic limits cannot be understood in disjoint terms, although they can be studied separately. Therefore, evidently, each limit can have a double interpretation: 1) a specific reading that distinguishes the logical limits and the paradigmatic limits of scientific knowledge; 2) a general reading that considers the limits of scientific knowledge as a whole with elaborative and representative features. In any case, the concept of limit should not suggest the idea of something insurmountable. In fact, the logical and paradigmatic limits of scientific knowledge determine the field of extension of the acquiring capacities. The acquisition of new knowledge is limited by certain styles of rational elaboration and by certain patterns of world representation. What is beyond these limits is not covered and it cannot be acquired. But this condition marks the possible transition of science to phases of change or to real revolutions, as evidenced by Kuhn. In fact, the new discoveries and the new acquisitions can cause the breaking and the overtaking of the dominant limits, so developing new logical forms and new paradigms that inevitably will develop new limits. Hence, we can understand the nature of the limits of scientific knowledge as being, at the same time, both rigid and elastic.

In what follows, we will focus our efforts especially on logical forms, on logical limits and their foundations. The arguments related to these matters are, in fact, relatively new, and so we believe that their deeper exposure is at least useful, if not necessary.

2. The Logical Forms of Scientific Knowledge

From a historical point of view, scientific knowledge takes different logical forms. We can operationally define the logical form in the terms of a structure, a configuration, or a rational style that scientific knowledge can take in its history.

1) We call the first logical form *Strong Deductivism*. It is based on an axiomatic logic inspired by an idealistic philosophy. It is typical of the more systematic tendencies of science that plunge into Euclidean Geometry and take expression in modern formalist approaches. The theoretical model of strong, or exclusive, deduction implies the massive intervention of the operative and instrumental mental mechanisms. Deduction, lacking of investigative qualities, is a simple

application to reality of the aprioristic and pre-existing certainties that are irrelevant to experiences and to learning. The mind explains the reality in a rational way, by identifying some of its well-established truths. This theoretical setting translates deduction into a recognition mechanism of the world. It is exclusive because it excludes all the mental processes aimed at discoveries. It is strong because, from its point of view, deduction has no need of anything more than itself to be complete.

2) We call the second logical form *Weak Deductivism*. It is based on a formal logic inspired by an empiricist philosophy. This is typical of the positivistic and neo-positivistic tendencies of science that are impressed by the discoveries of contemporary quantum physics. The theoretical model of weak, or inclusive, deduction lightens the burden of innate ideas. It refers the certainty of knowledge to a hard, complex and demanding act of study, ideation and investigation of the reality. So, knowledge evolves gradually: through the ages, it avails itself of amazing works of genial personalities able to enlarge the mass of knowledge through their intuitions and clarifications. Deduction is a part of the largest process of rationality and a conclusive phase aimed to tidy up all knowledge, already learned through a hard inductive work of investigation starting from available data in reality. Deduction has the characteristics of a knowledge accommodation mechanism. It is strongly inclusive because it needs the intervention of mental processes aimed at axioms, discoveries, and creations. It is weak because, from its point of view, deduction needs something more than itself to be complete.
3) We call the third logical form *Pure Inductivism*. It is based on a hypothetical logic inspired by an infinitary philosophy. This feature is typical of all the relativistic tendencies of science that reject the absolute value of knowledge. In fact, knowledge has no boundaries and no limits. No mechanism is able to stem its endless flow. This continuity is unstoppable and not bound in a delimited and discrete field of knowledge. Calculability loses its value and at the same time the infinite dilution of all meanings dominates all. Induction flows pure, without obstacles. Inductivism sets the idea of inexhaustible empiricism as its implicit axiomatic base. In this logical form, empiricism is an axiom of inexhaustibility.

Each of these trends implies some logical limits that are evident in paradoxes and that arise from their own philosophical basis.

3. Strong Deductivism

Axiomatic logic is typically deductive. It is structured around theoretical certainties and it uses closed methodologies. Reasoning aspires to the perfection of its knowledge systems, avoiding an infinity reference of demonstrations and theorizations.

Each perfect theory explicates irrefutable truths in following these criteria:

- Argumentative expressivity that shows all the contents.
- Decidability of method that verifies or confutes all the statements using algorithmic modalities.
- Coherence of reasoning that emerges from brief statements without a contradiction.
- Completeness of judgements that includes all the possible truths of a particular argument.

David Hilbert is the greatest exponent of this idea in 20th century. Thanks to him, a new logical tendency rises: Formalism. He proposes the idea of closed theoretical systems structured around a finite numbers of symbols and symbol relations. These systems are able to represent knowledge in an exhaustive way.

In 1900, during the "Second International Congress of Mathematicians in Paris," Hilbert presents a famous dissertation about contemporary history of mathematics. He proposes a list of 23 problematic questions. The second of these questions asks to demonstrate the coherence of arithmetic. Hilbert wants to avoid the risk of having logical contradictions. Coherence becomes a necessary parameter of demonstrative rigor. Few years later, in 1928, during the "International Congress of Mathematics in Bologna," Hilbert[1] takes on the ambitious task of giving to mathematics a full and final formal shape, in fixing it as a homogeneous and structured system. He thinks to a mechanical procedure able to resolve all mathematical problems and that belongs to a defined symbolic class.

Hilbert[2] challenges the mathematical academic world, starting from his concerns about non-Euclidean[3] geometries and paradoxes, factors of an undoubted logical weakness. So, he believes that is necessary a complete formalization of the whole discipline. In this way, mathematics would be reduced to a pure system of

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[1] David Hilbert, "Problemi della fondazione della matematica," in *Ricerche sui fondamenti della matematica*, ed. V. Michele Abrusci (Naples: Bibliopolis, 1978), 291-300.

[2] David Hilbert, "Nuova fondazione della matematica. Prima comunicazione," in *Ricerche sui fondamenti della matematica*, 189-213.

[3] The non-Euclidean geometries arise in reaction to the Euclid's fifth postulate, the only one more intuitive (Thomas L. Heath, *The Thirteen Books of Euclid's Elements* (New York: Dover, 1956)).

axioms, symbols, formulas, rules and demonstrations. In fact, for Hilbert, the paradox found in some logical formulations is due to their semantic value. Logic and mathematics are 'polluted' by the reference to meaning of words and of represented things. Therefore, he believes that is necessary to create a meaningless frame. So any statement about a system becomes the product of meta-mathematical observations performed in according to a language that is outside of the mathematical language itself. So, we can advance statements about the characteristics of the mathematical system with the use of the terms and the rules of another language. For example, if we admit that the formula "$x+0=0$" is a theorem of the formal arithmetical system, then we are not talking about the mathematical semantics, but we are talking about a syntactic characteristic of mathematics in the terms of a language (english) that is different from the mathematical one. For Hilbert,[4] meta-mathematics is outlined like a new branch of mathematics, interested in what it is possible, or not, to demonstrate.

According to Hilbert, a formal mathematics respects coherence, completeness, expressivity and decidability. In this way, the non-Euclidean presuppositions would collapse and the risk of a not delimited infinity would be avoided. Hilbert's model is, in fact, finite because it loses the inconvenient concept of infinity in act.[5] It follows the regular flow of its demonstrative methods.[6] Mathematics must not refer to the actually infinity objects, but only to the potentially infinite collections as the natural number sequence (N). The problem of foundations[7] originates when reason goes beyond abstraction and idealization limits, arriving to the transcendental notions that lead to paradoxes and not empirical geometries. The idea of the infinity is uncertain, fleeting and not theorizing. When the semantic infinite drift is established[8] and meta-mathematics is limited to a syntactic level, mathematics would be anchored to a formal and symbolic finitude. So, a

[4] David Hilbert, "I fondamenti logici della matematica," in *Ricerche sui fondamenti della matematica*, 215-31.

[5] The distinction between the actual infinity and the potential infinity dates back to Aristotle. The first term indicates that the infinity is something active. The second term considers the infinity as a purely cognitive instrument (Jeanne-Pierre Luminet, Marc Lachiéze-Rey, *De l'infini… Mystéres el limites de l'Univers* (Paris: Dunod, 2005)).

[6] David Hilbert, "Sull'infinito," in *Ricerche sui fondamenti della matematica*, 233-66, "Conoscenza della natura e logica," in *Ricerche sui fondamenti della matematica*, 301-11.

[7] The dispute around the mathematical foundations arises between the 19th and the 20th centuries.

[8] David Hilbert, "La fondazione della Teoria Elementare dei Numeri," in *Ricerche sui fondamenti della matematica*, 313-23.

mathematician must respect the *Finiteness Theorem* that ensures the exclusive derivation of the theoretical consequences from some finite axiomatic premises.

For Hilbert,[9] axiomatic logic is an unrepeatable chance for the growth of science. It facilitates the overtaking of all the epistemological doubts around foundations in referring to sign and formalization.[10] Moreover, it prevents the contradictory that is, instead, typical of immature sciences.[11] So, each symbolic and axiomatic system represents a closed logic.

This sort of logical closure brings some undoubted advantages: 1) Simplicity. The limited vocabulary of terms is a good instrument and as a linear method is a good procedure;[12] 2) Conceptual representation. Each idea is an evident certainty. It takes concreteness and it becomes a clearly element even if only symbolic;[13] 3) Essentiality. All ideas and statements of a formalized system are logical nuclei that create a complicate and interactive matrix.[14] But, beyond the advantages, closed logic leads to a no good knowledge fragmentation. Formalism produces, in fact, several logical systems that respond to perfect prerequisites.

3.1 The Idealistic Limits of Strong Deductivism and the Open Logic

Kuert Gödel[15] proves that arithmetic cannot be completely formalized. The *Arithmetical Incompleteness Theorems* establish that any closed formal system, independently from its breadth, is necessarily incomplete, because any statement concerning its internal character cannot come from its own language, but from another one, outside of it. Gödel's discovery shows how any mathematical system, even if apparently complete and correct, includes some statements that are not

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[9] David Hilbert, "Pensiero assiomatico," in *Ricerche sui fondamenti della matematica*, 177-88.

[10] David Hilbert, "Sui fondamenti della logica e dell'aritmetica," in *Ricerche sui fondamenti della matematica*, 163-75, "Nuova fondazione della matematica," "I fondamenti della matematica," in *Ricerche sui fondamenti della matematica*, 267-89.

[11] David Hilbert, "Problemi matematici," in *Ricerche sui fondamenti della matematica*, 145-62, "Pensiero assiomatico," "Nuova fondazione della matematica," "Dimostrazione del Tertium non Datur," in *Ricerche sui fondamenti della matematica*, 325-30

[12] Hilbert, "Problemi matematici."

[13] Hilbert, "Pensiero assiomatico."

[14] Hilbert, "Pensiero assiomatico," "Conoscenza della natura," David Hilbert and Paul Bernays, "Grundlagen der Mathematik," in *Ricerche sui fondamenti della matematica*, 329-474.

[15] Kurt Gödel, "On Formally Undecidable Propositions of Principia Mathematica and Related Systems I," in *Kurt Gödel. Collected Works Volume I: Publications 1929-1936*, eds. Solomon Feferman, John W. Dawson Jr., Stephen C. Kleene, Gregory H. Moore, Robert M. Solovay, and Jean van Heijenoort (New York: Oxford University Press, 1968), 144-95. The first public formulation of Gödel's theorems dates back to October 7, 1930, during a conference in Königsberg.

demonstrable or refutable by its own logical tools. The truth of these statements is not resolvable through the procedures allowed by the system. Gödel discovers that deductive logic does not consent to prove all the possible true relationships between the numbers. The truth always goes beyond demonstrations. Some propositions cannot be proven true. Formalism is thus destroyed.

In the attempt to resolve the second great question of Hilbert's list, Gödel[16] studies arithmetical coherence and the problems that it entails. He is soon faced with the extreme certainties of Formalism. Anti-coherence, or paradox, is a special self-referral idea. When a system states something about itself, it produces irresolvable statements, on which it is not possible to demonstrate truth or falsity. Thanks to Gödel, mathematics talks about itself and so it let itself to fall into a logical self-reference.

A formalized mathematical system is, at the same time, a set of uninterpreted symbols and a set of linguistically interpreted symbols. In accord with Hilbert, Gödel distinguishes the object-language of numbers and the meta-language of the ordinary words that describes the same numbers. But, differently from Hilbert, he links theory and meta-theory. So, if numbers are, at the same time, terms of object-language and meta-language, then mathematics talks about itself and it becomes a self-referral system.[17] Gödelization, or Gödel's numbering,[18] is the procedure that allows the mathematical self-reference through the arithmetization of the meta-theory. It is an ingenious method that transforms logical variables into numerical variables.

In *Principia Mathematica* of 1910-1913, Alfred North Whitehead and Bertrand Russell[19] create a complex logical system that represents arithmetic as a good meta-language. In this model, the symbols express syntax, in talking about numbers and their relationships.[20] Gödel translates this logical system into an arithmetical system. Each of its logical entities is transformed, in fact, into a number, so at the end numbers will talk about other numbers. Formulas and logical demonstrations are associated with particular arithmetical notions (Gödel's

[16] Kurt Gödel, "On the Completeness of the Calculus of Logic," "The Completeness of the Axioms of the Functional Calculus of Logic," "On the Completeness of the Calculus of Logic," "Some Metamathematical Results on Completeness and Consistency," in *Kurt Gödel. Collected Works Volume I*, 61-101, 102-23, 124-5, 140-3.

[17] Gödel, "On Formally Undecidable Propositions of Principia Mathematica."

[18] The technique is also known as Diagonalization, Syntax Arithmetization or Fixed Point.

[19] Alfred N. Whitehead and Bertrand A. W. Russell, *Principia Mathematica* (Cambridge: Cambridge University Press, 1962).

[20] Gödel, "The Completeness of the Axioms," "On Formally Undecidable Propositions of Principia Mathematica," "On the Completeness."

numbers). This method gives a natural number to each discrete object of a logical system. Any symbol (ε), formula or string receives its Gödel's number, or Gödelian $g(\varepsilon)$. Gödelization gives various numbers to various expressions, so it is also an algorithmic procedure. If we have any expression (ε), then we can immediately calculate its number $g(\varepsilon)$, and if we have any natural number (n), then we can immediately identify an expression (ε) for which $n=g(\varepsilon)$.

Thanks to this self-reference, Gödel discovers the logical paradox of any system based on numbers. He translates Epimenides' revised form of the classical Eubulides' paradox, "This sentence is false" in the assertion "This sentence is not demonstrable." So, he translates this new assertion in numerical terms (Gödelian statement or *G*). If it is demonstrable then it is true, but based on what it says, it is not demonstrable. On the contrary, if it is not demonstrable then it is false, but contradicting what itself says, it is demonstrable. Thus, mathematics is opened to logical paradoxes and it cannot be entirely coherent and complete.[21]

So Gödel formulates his two famous theorems that, after some years,[22] he will describe in informal way as his most important logical discoveries. On the basis of the *First Incompleteness Theorem* (*G1*), we can affirm that, in the language (*L*) of a correct mathematical system (*S*), an undecidable statement (*G*) inevitably arises. *G* is not demonstrable or rebuttable within *S* and with the language of *S* (*L*). The completeness of a mathematical system is not achievable. Instead, on the basis of the *Second Incompleteness Theorem* (*G2*) that derives from the first, we can affirm that if *S* is a correct mathematical system, then *S* will not prove its internal coherence. This theorem causes the final collapse of any formalist illusion and it gives a negative answer to the question on coherence presented by Hilbert in Paris. We cannot prove the coherence of any logical system if we are into the same system.

An interesting generalization of Gödel's discoveries comes from the developments of logic and from his same statements.[23] During thirties, Alan

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[21] Gödel, "On Formally Undecidable Propositions of Principia Mathematica," "Lecture on Completeness of the Functional Calculus," in *Kurt Gödel. Collected Works Volume III: Unpublished Essays and Lectures*, eds. Solomon Feferman, John W. Dawson Jr., Warren D. Goldfarb, Charles D. Parsons, and Robert M. Solovay (New York: Oxford University Press, 1995), 16-29, "On Undecidable Sentences," in *Kurt Gödel. Collected Works Volume III*, 30-5.

[22] Kurt Gödel, "Some Basic Theorems on the Foundations of Mathematics and their Implications," in *Kurt Gödel. Collected Works Volume III*, 304-23.

[23] Kurt Gödel, "Postscript to Spector 1962," in *Kurt Gödel. Collected Works Volume II: Publications 1938-1974*, eds. Solomon Feferman, John W. Dawson Jr., Stephen C. Kleene, Gregory H. Moore, Robert M. Solovay, and Jean van Heijenoort (New York: Oxford

Turing's works help and support Gödel's Theorems. Turing proposes a good solution to the definition of calculability, so reinforcing Godel's *Incompleteness Theorems*. In fact, decidability, which is the base of these theorems, is associated with the idea of calculability. So, a lack of definition of this idea is a weakness. Turing[24] idealizes a calculating machine and he defines "calculation" all the process that this machine makes. Calculation is intended as the ability to keep in mind a set of rules. It is an algorithmic, mechanical and linear procedure. Turing's Machine (TM) is characterized by finite sets of states and instructions that move a reading and writing head along the compartments of a potentially infinite tape. Gödel thinks that the TM is a rigorous and adequate demonstration of the notion of effective procedure. Above all, thanks to this mechanical model, the idea of incompleteness regards each formal system defined by the mechanical production of theorems. A formal system is constituted by rules that transform some statements in other statements and that can be directly applied by a human agent or indirectly incorporated and executed by a machine. The necessity of arithmetization disappears and the phenomenon of incompleteness seems really belong to each mechanized method. Gödel[25] admits that his theorems are applicable to all formal-logic systems and not only to mathematical ones. Each logical system must be syntactically and semantically opened.

Gödel[26] demolishes the strong idea of Formalism and develops a sort of open logic that surpasses the idea that man can know irrefutable scientific truths. Truth is only partial and perfectible. For Gödel,[27] a formal system cannot be closed and it cannot entrench itself in certainties that are held as always valid. The universe of ideas transcends human knowledge.

The idealistic limitation that Gödel imposes to Formalism is, nearly, confirmed by the results of other important logicians. Alfred Tarski[28] defines the

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University Press, 1990), 253, "What is Cantor's Continuum Problem?" in *Kurt Gödel. Collected Works Volume II*, 254-70.

[24] Alan M. Turing, "On Computable Numbers, with an Application to the Entscheidungsproblem," *Proceedings of the London Mathematical Society* s2-42, 1 (1937): 230-65, "On Computable Numbers, with an Application to the Entscheidungsproblem. A correction." *Proceedings of the London Mathematical Society* s2-43, 1 (1938): 544-6.

[25] Gödel, "Postscript to Spector," What is Cantor's Continuum Problem?"

[26] Gödel, "Some Basic Theorems."

[27] Gödel, "On a Hitherto Unutilized Extension of the Finitary Standpoint," in *Kurt Gödel. Collected Works Volume II*, 240-1, "On an Extension of Finitary Mathematics which has not yet been Used," in *Kurt Gödel. Collected Works Volume II*, 271-80.

[28] Alfred Tarski, *Pojecie Prawdy w Jezykach Nauk Dedukcyjnych* (Warszawa: Nakladem Towarzystwa Naukowego Warszawskiego, 1933), "The Concept of Truth in Formalized

syntactic and semantic opening of a formal system in terms of the linguistic higher order evaluation. Tarski creates the *Undefinability Theorem* according to which arithmetic cannot express and define some numbers properties, as the concept of truth. So, arithmetic loses some contents and the formalist assumption of argumentative expressivity. Finally, Alonzo Church[29] and Barkley Rosser[30] create the *Undecidability Theorem*. It establishes that, starting from some theoretical axioms, we cannot create a general algorithm that is able to verify, or not, each specific logical content. With the further collapse of logical decidability myth, Hilbert's proposal to axiomatize mathematics and science in general is not realizable.

Gödel's results reveal that the limits of axiomatic models come from the excesses of rigor. These are signs of reductive mentalities conditioned by the myth of syntax and the underestimation of semantic.[31] The man cannot create correct and complete theories of universe.[32] Science is the final judge of truth, but it must also recognize and elaborate its limitations.[33]

3.2 The Foundations of Open Logic

Beyond non-Euclidean geometry, a revolutionary moment in the history of logic is the discovery of Russell's paradoxes that confute the set theory of Gottlob Frege. In a first moment, Frege and Russell refuse Giuseppe Peano's theoretical system[34] because of its fragility, its unclarity in demonstrative steps and real priority of its

Languages," in *Logic, Semantics, Metamathematics*, ed. John Corcoran (Indianapolis: Hackett, 1983), 152-278.

[29] Alonzo Church, "An Unsolvable Problem of Elementary Number Theory," *American Journal of Mathematics* 58, 2 (1936): 345-363, *Introduction to Mathematical Logic* (Princeton: Princeton University Press, 1956).

[30] Barkley J. Rosser, "Extension of Some Theorems of Gödel and Church," *Journal of Symbolic Logic* 1, 3 (1936): 87-91, "An Informal Exposition of Proofs of Gödel's Theorem and Church's Theorem," *Journal of Symbolic Logic* 4, 2 (1939): 53-60.

[31] R. Rosen, "Undecidability and Biology," in *On Limits*, eds. John L. Casti and Joseph F. Traub (Santa Fe Institute: Teleconferences (Report 94-10-056), 1994), 15-6.

[32] Stephen Hetherington, "Knowledge's Boundary Problem," *Synthese* 150, 1 (2006): 41-56.

[33] Christopher Cherniak, "Limits for Knowledge." *Philosophical Studies* 49, 1 (1986): 1-18.

[34] In 1889, Peano (Giuseppe Peano, "The Principles of Arithmetic, Presented by a New Method," in *A Source Book in Mathematical Logic*, ed. Jean van Heijenoort (Harvard: Harvard University Press, 1967), 83-97) tries to axiomatize the arithmetic, in retracing the Euclidean geometrical model. Primitive concepts: zero; number; successor. Axioms: 1 is a number; the successor of a number is another number; if two numbers are equal, then they have the same successor; 1 is not the successor of another number; if k is a set, 1 is part of k and, for each number (x), if x is part of k, and also $x+1$ is part of k, then k contains the entire class of N.

basilar concepts. So, these two authors inaugurate the logicist program proposing to bring mathematics to its logical and primary bases. This is also a rigid axiomatization that is, however, focused on the aprioristic set theory. Frege and Russell sustain that sets are more primitive concepts than numbers. In fact, numbers are always definable in terms of sets. Each number can be reduced to a most primordial notion that is only guess. The *Theory of Complex Sets*, created by Frege,[35] postulates that each mathematical element can be included in a specific set that is, also, a part of a wider set. This is a closed structure where we can insert any mathematical statement. There are sets that refer to other sets and that close everything in general axiomatic wholes that are, in turn, confined into more extensive sets. At some point, concatenation should end with the absolute wider set that can include each other set, even itself. But this epilogue represents the maximum limitation of this closed logical formulation.

The same Russell[36] discovers a deep antinomy inside this set theory. *R* is the set of all sets not members of themselves. *R* is a collection of sets, so an *x*-set can be part of it if and only if it is not a member of itself. But is *R* a member of itself, or not? If it is not a member of itself, then it is part of *R*, because *R* is constituted, to definition, from all sets not members of themselves. In this case, *R* is part of itself, but this affirmation contradicts the early statement. Instead, if *R* is a member of itself, then it is a part of sets not members of themselves; *R* is not a part of itself.[37]

In *Principia Mathematica*, Russell tries to save the Logicism from paradoxes, elaborating the *Logical Types Theory*, which Gödel[38] well describes. Russell argues that each set is part of a level that is higher than the level that includes its same constitutive elements. So, the concept of a set that includes all sets is a mistake. Russell renounces to the idea of sets for the new idea of types that assumes two different versions: a simple one and a complex, or branched, one. Paradoxes arise from a vicious circle consisting in the supposition that a collection of objects can contain members whose definitions derive from the collection intended as a whole. To avoid this logical self-reference of totality, or set, to itself we must formulate some statements not included inside its range of references.[39] The paradox of sets arises from the belief that all sets are part of the same type.

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[35] Gottlob F. L. Frege, *The Basic Laws of Arithmetic* (Berkeley: University of California Press, 1964).

[36] Bertrand A.W. Russell, "Letter to Frege," in *From Frege to Gödel*, ed. Jean Van Heijenoort (Cambridge: Harvard University Press, 1967), 124-5.

[37] *X* is a class. R is a set of collection defined as $(X \in R) \leftrightarrow (X \notin X)$. But $X=R$, because *R* is however a class. So, *R* is defined as $(R \in R) \leftrightarrow (R \notin R)$.

[38] Kurt Gödel, "Russell's Mathematical Logic," in *Kurt Gödel. Collected Works Volume II*, 119-43.

[39] Bertrand A.W. Russell, *The Philosophy of Logical Atomism* (Peru: Open Court, 1985).

Whereas, we must understand that the properties of a higher type are applied, or predicate, only to the objects of a lower type.[40]

Thanks to Russell and Gödel's works, we can know the inexhaustibility character of open logic. In fact, if we decide to add the Gödelian undecidable statement (*G*) among the axiomatic system, so creating a more powerful system, however the new system will have its own undecidable statement. So, we can consider the formula "$S=\gamma$", where S is a system identified with its undecidable statement (γ). Then, we can consider γ as an axiom of another powerful system (S_1). But also in this case, the presence of another undecidable statement (γ_1) would be proved: $S_1=(S+\gamma)+\gamma_1$. We can proceed further and the results would delineate a regularity such as $S_2=(S_1+\gamma_1)+\gamma_2$, $S_3=(S_2+\gamma_2)+\gamma_3$ $S_4=(S_3+\gamma_3)+\gamma_4$....., $S_n=(S_{(n-1)}+\gamma_{(n-1)})+\gamma_n$. At a certain point, the mind cannot go beyond. If a formal system indefinitely postpones its undecidability, then the human mind could not see its coherence. Everything would turn into its origin ($S_\infty=\gamma_\infty$) and confirms the undecidability of the infinity.[41]

The logical infinity is an essential aspect of human knowledge. It doesn't set limits, but it exalts the fallibility of each certainty that is limited to closed fields. Infinity delays to not delineable horizons and causes the end of all cognitive illusion of infallibility.

4. Weak Deductivism

The empirical perspective considers and preserves data without transcending their concrete level:

1) The objectivity of the observation aspires to capture data, without any subjective interference. The observer must dismiss his subjectivity, becoming a mechanical container of external occurrences.

2) The certainty of reason is structured on objective data. Thanks to an inductive procedure, the mind elaborates any fact and the totality of them to formalize some certain principles. These principles are similar to the idealistic axioms but different because they come from the experiences.

This empirical logic finds fruitful ground among the followers of neo-Positivism. The theorists of Vienna Circle are the most important exponents of this perspective. They were a group of scholars interested in an objective evaluation of reality and in abjure any speculative tendency. Philosophy must

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[40] Russell, *The Philosophy of Logical Atomism*,

[41] Gödel. "Lecture on Completeness," "The Present Situation in the Foundations of Mathematics," in *Kurt Gödel. Collected Works Volume III*, 45-53

leave the worldviews elaboration to concentrate itself on a conceptual clarification. Real scientific success is inside the encounter between empirical observation, which constitutes the objective base, and the subsequent mathematical deduction.[42] In accord with Auguste Comte's positivism and with the empiricism of modern age, neopositivists think that knowledge must be based on scientific experience and it must be explicated thorough a symbolic logic.[43] In a formalistic way, science can take a precise and formal representation. Logical neo-positivism distinguishes scientific phase of discovery and scientific phase of justification. The first one is a non-logical elaboration of hypotheses, because discovery doesn't possess effective rules.[44] The second one, however, is a purely evaluation of the same hypotheses, because it merges data and theories.[45] Thus, some knowledge courses become indispensable: 1) signification of scientific terms; 2) nomological and deductive explication; 3) hypothetical and deductive justification; 4) theoretical axiomatization. The verification of meaning follows the perfection of terminology definition,[46] to avoid the risk of a conceptual confusion. The meaningless scientific assertions are not false, but they are incomprehensible. So, observational propositions become essential. Their value of truth derives from sensorial immediacy. These propositions represent the states of the physical world, in reducing observation to a realistic physicalism. The other propositions, which express some unobservable concepts (ex. physical strength), can be indirectly verified from the same observable propositions. The new knowledge must be reduced to the symbols used to represent the old knowledge.[47] Percy Bridgman's Operationism,[48] for example, leads us to consider each new scientific term contained into some propositions that can be confirmed or confuted thorough some operations. When the meaning of the term is established, we can apply deductive explication to formulate some schematic explications, typically mathematical and nomological.[49] These explications derive from the known statements that are both primary, because they express scientific laws, and secondary, because they describe empirical facts intended as conditions of knowledge. Laws are conditional assertions ("if *x* happens, then *y* happens"),

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[42] Hans Reichenbach, *The Rise of Scientific Philosophy* (Berkeley: University of California Press, 1951).
[43] William Bechtel, *Philosophy of Science* (Nillsdale: Lawrence Erlbaum, 1988).
[44] Reichenbach, *The Rise of Scientific Philosophy*.
[45] Reichenbach, *The Rise of Scientific Philosophy*.
[46] Moritz Schlick, *Forma e contenuto* (Turin: Bollati Boringhieri, 2008).
[47] Schlick, *Forma e contenuto.*
[48] Percy W. Bridgman, *The Logic of Modern Physics* (New York: Beaufort, 1927).
[49] Schlick, *Forma e contenuto.*

which exclude any exception.[50] The initial conditions affirm the prior fact's occurrence ("*x* has happened"). So explication becomes, through *modus ponens*, a clear deductive conclusion (then *y* happens). Hypothetical and deductive method becomes necessary to identify scientific laws. Starting from an event that needs clarifying explications, scholars propose some verifiable hypotheses on the base of a continuous testing. Therefore, certain hypotheses, which are inductive and probabilistic processes of data enumeration,[51] are compared to scientific laws.[52] Finally, the strong axiomatic perspective of all the logical neo-Positivism takes evidence and it reduces empirical dimension to a closed system. In contrast to its same initial intentions, Neo-Positivism follows a Euclidean logic. In fact, when it affirms that a logical law is explicated, it reduces it to a reference theory. At the same way, Neo-Positivism explicates the events, in reducing them to a logical law.[53] This is a closed and circular course in which theories are deductive structures with primitive terms and these axioms assumes a logical laws form. Science must be divided in forms and structures, thus to be quantifiable.[54]

Empirical and axiomatic logic is objective and certain, limited and obstructed. Therefore it is a closed logic. This logic doesn't differ from deductive and exclusive logic, that is immolated to the myths of coherence and completeness. According to logical empiricism, induction is a rational searching of truths that uses intuition to recognize the theoretical primacy of each concept.[55] The true axioms are always evident, enveloped and limited.

Weak Deductivism associates axioms and experiences and so, in this perspective, rigidity has a different meaning. This logic doesn't neglect the possibility of changes, because it considers axiomatization an ultimate target of certainty and not something already known. This is a challenging course that consists of difficult confirmations, but also of many refutations. The meaning of *Weak Deductivism* emphasizes that a procedure cannot betray the value of each human knowledge experience. The idea of axiomatic perfection predisposes this logic to closure, losing the opportunity to consider its own opening.

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[50] Reichenbach, *The Rise of Scientific Philosophy.*

[51] Carl G. Hempel, "Studies in the Logic of Confirmation I," *Mind* 54, 213 (1945): 1-26, "Studies in the Logic of Confirmation II," *Mind* 54, 214 (1945): 97-121, *Aspects of Scientific Explanation and other Essays in the Philosophy of Science* (New York: Free Press, 1965), Reichenbach, *The Rise of Scientific Philosophy*, Schlick, *Forma e contenuto.*

[52] Schlick, *Forma e contenuto.*

[53] Bechtel, *Philosophy of Science.*

[54] Schlick, *Forma e contenuto.*

[55] Reichenbach, *The Rise of Scientific Philosophy.*

During 20th century, this logic has other estimators: 1) the Bourbaki Group[56] believes that any axiomatic system must be proved on the base of experience; 2) Haskell Curry,[57] a founding father of combinatory logic,[58] thinks that formal modality is the most important aim of any logical system. Each logical structure can be considered valid until it doesn't contrast with another data that activate the confutation process and a subsequent reformulation; 3) Saunders Mac Lane[59] considers each axiomatic system as an intuitive result of human activities that assumes theoretical consistency and, at the same time, it deviates away from these same activities. In these scholars, axiomatization encounters an empirical foundation and it acquires mutability and flexibility. However, it doesn't lose its essential narrowing quality.

4.1. The Empiricist Limits of Weak Deductivism and Hypothetical Logic

Inclusive approaches fall in a logical closure. In an attempt to contrast idealistic abstractionism, these approaches assume an empiricist perspective without detracting the domain of deductive method. In fact, when the axioms are objectively identified, the subsequent deductive rationalization remains intact. But, according to Jackson,[60] if two of the most important limits of knowledge arise from reason constraints and from relationship between the subjective world and the objective one, then inclusive empiricism is doubly limited. In fact, it follows both axiomatic perfection and empirical certainty.

The objectivity of observation is the first focal concept of this type of logic. The term 'objective' refers to what is concrete, because it has not subjective implication. An evident contradiction is already inherent: the exclusive relationship between observation and objectivity. Anyway, observation is always a human act, a subjectivity product. The same assertion can be valid for the observational instruments that can replace human acts, in introducing more assurances of validity, but that remain always human creations. The act of observation cannot be totally objective because it is, above all, a subjective act.

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[56] Nicolas Bourbaki, *Eléments de Mathématique* (Paris: Hermann, 1939).

[57] Haskell B. Curry, *Outlines of a Formalist Philosophy of Mathematics* (Amsterdam: North Holland, 1951), *Foundations of Mathematical Logic* (Mineola: Dover, 1979).

[58] Haskell B. Curry, Robert Feys, *Combinatory Logic I* (Amsterdam: North Holland, 1958), Haskell B. Curry, J. Roger Hindley, and Jonathan P. Seldin, *Combinatory Logic II* (Amsterdam: North Holland, 1972).

[59] Saunders Mac Lane, *Mathematics, Form and Function* (New York: Springer-Verlag, 1986).

[60] E. Attlee Jackson, "Final Comments in the Workshop Limits to Scientific Knowledge," in *On Limits*, 18-9.

An important confirmation to the paradox of the objectivity comes from the quantum physics of the 20th century. Werner Heisenberg[61] formulates the *Uncertainty Principle*, an important inducement for contemporary philosophical reflections. This principle postulates that quantum mechanics escapes from the correct measurements. For example, to calculate the position and the velocity of an electron inside the atom, we must illuminate it. But, in this way, the electron is struck by the photon and, because the Compton's effect, it changes position and velocity. If we decide to decrease the intensity of light and, as consequence, the emissions of the photons, then the velocity of electrons will be less disturbed, also if it is more difficult to identify their positions. The position and velocity of electrons cannot be simultaneously measured. A good knowledge of one of these two values presupposes the impossibility of knowing the second value. The observational act affects the observed reality, in producing inevitable interferences.[62] We don't observe the pure nature of the object, but this nature conditioned by the observational methods. The objectivity of observation is a scientific utopia.[63] At this point, each theory is lawfully adaptable to the limits imposed by objectivity. In this way, we can avoid the inconvenient questions, because if there are differences between a theory and the real world,[64] then only the theory is influenced by the limits of knowledge.[65]

Regarding the observational instruments, we can go further back in time and remember the intention of the astronomer and mathematician Friedrich Wilhelm Bessel[66] to focalize the attention on the systematic errors made by researchers during the measuring of stars position. Bessel notes that is important to consider the difference between the apparitions of the astronomical phenomena, their visual perception and the subsequent measurement of reaction done by researchers. The latency may explain the final measurement errors. During this period, several individual variations intervene as reflection of many psycho-physiological conditions (fatigue, tiredness and attention decline). Bessel

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[61] Werner K. Heisenberg, "Ueber die Grundprincipien der Quantenmechanik," *Forschungen und Fortschritte* III (1927): 83.

[62] Werner K. Heisenberg, *Physics and Philosophy* (New York: Harper, 1958).

[63] Heisenberg, *Physics and Philosophy*.

[64] James B. Hartle, "Scientific Knowledge from the Perspective of Quantum Cosmology," in *Boundaries and Barriers: On the Limits to Scientific Knowledge*, eds. John L. Casti and Anders Karlqvist (Reading: Addison Wesley, 1996), 117-47.

[65] R. Rosen, "On the Limitations of Scientific Knowledge," in *Boundaries and Barriers: On the Limits to Scientific Knowledge*, 199-214.

[66] Simon Schaffer, "Astronomers Mark Time: Discipline and the Personal Equation," *Science in Context* 2, 1 (1988): 101-131.

introduces the concept of *Personal Equation of the Observer* to measure the interferences of those variables.

Exactness is another central concept of inclusive perspective. It is a quality of perfection that should crown any inductive effort of the reason. But, there are several types of induction in logic and each of them has its limits.

Associative induction, for example, is incomplete. In fact, it produces conclusions based on partial similarities between observed data. The intellect finds some common traits between two entities and so it infers the presence of another trait of similarity: if *A* and *B* are similar in an aspect (*m*), then a similarity in another aspect (*n*) is inducible. This is a shared association of elements that becomes a minimal and inconclusive generalization. This induction assumes its maximum validity only when the knowledge of data is extended, the similarities are considerable and the differences are very few.

Enumerative induction, instead, is a generalization that moves from some facts to extended conclusions. This is a pure numerical procedure that formulates certain conclusions on the basis of data that confirm them. Evidently, a certain empirical induction should be confirmed by a complete enumeration of all observable data. This condition, however, is valid only in cases of reduced sets. In other cases, instead, enumerations are incomplete and subsequent conclusions are partial and always refutable when a new fact come to contrast the facts previously observed. In these cases, enumerative induction is not certain but it is probable as well as associative induction. It is, in fact, interpretable in a statistical meaning. So, each empirical proposition assumes a hypothetical value.

Finally, causal induction goes back from known effects to their unknown causes. This procedure cannot be completely delineated. The way to know the causes can start from the observation of the constancy of some effects or from the observation of an exception in the regularity of the same effects. Also in this case, the research of causes requires a hypothetical attitude that links observations, experiments, possibility and validity. In any case, the causes may remain obscures, and so only the possible causes must be preserved.

Induction is always hypothetic both when is associative and empirical, thus probable and quantitative, and when is causal, thus possible and qualitative.[67] Knowledge shows clear empirical limits that prevent the confirming hypothesis

[67] While the analogical and enumerative inductions are based on probable hypotheses, the causal ones are based on possible hypotheses. The possibility refers to what could happen to a purely qualitative level. The probability refers, instead, to what could happen to a quantitative and statistical level.

correction. Above all, the improbable hypothesis that everything is knowable is simply unacceptable.[68]

4.2. The Foundations of Hypothetical Logic

The empirical investigation of reality falls into some paradoxes, when it is founded on objectivity and on certainty presuppositions. Detached scientific experience of the world, which transforms scientist in a cold machine, doesn't permit the perfect knowledge, but only an illusion of knowability. Observation will be always determined by what it comes from the same scientist. Observation of the world things means, primarily, bringing them into being. Even here, the boundary lines are dissipated in open horizons. The scientist, in his observational act, firstly brings and takes the same observation merged with the object of his interest. The observed reality is an interaction dimension in which we can do conjectures, speculations or real conclusions. But the risk is hidden inside this interaction. But without considering the contributions of Gestalt School regarding the illusions of perception, we find several paradoxes that arise from observation. For example, in the paradox of the solar eclipse,[69] the observer believes that is possible to see the dark side of the Moon, but what it is without light cannot be seen. Roy Sorensen[70] modifies this paradox. He invites us to imagine two different moons that make the paradox even more complex during a solar eclipse.

All knowledge is the summary of an essential coexistence often adorned with illusions and antinomies. The rigid and neutral observation cannot increase knowability and drags it to a utopian level of perfection in simply denying the existence of interactivity between object and subject. The negation of a nature and of its paradoxes causes the onset of other strong paradoxes. Frederic Fitch[71] proposes an interesting paradox of knowability. The basic idea concerns the existence of truths that are never completely knowable, although their possible conceivability. So, we can consider p as a proposition that expresses a not knowable truth and we cannot know its truth. On the contrary, p would become both a knowable and not knowable truth. Human knowledge is characterized by a certain ignorance that constitutes its premise.[72]

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[68] Cherniak, "Limits for Knowledge."

[69] Michael Clark, *Paradoxes from A to Z* (New York: Routledge, 2002).

[70] Roy A. Sorensen, "Seeing Intersecting Eclipses," *Journal of Philosophy* 96, 1 (1999): 25-49.

[71] Frederic B. Fitch, "A Logical Analysis of Some Value Concepts," *The Journal of Symbolic Logic* 28, 2 (1963): 135-42.

[72] Richard Routley, "Necessary Limits to Knowledge: Unknowable Truths." *Synthese* 173, 1 (2010): 107-22.

The weakness of inductive reasoning follows the illusion of objective observation and its subsequent knowability. Carl Hempel[73] is the creator of the famous paradox of the crows, a good example of the limitation of enumerative induction. In fact, a common generalization says that all the crows are black. However, this statement is not demonstrable in reality, because the set of crows is very dynamic and open, with a not delineable spatial and temporal extension. A real certainty is absent. Even a crow may be of a different colour, for example white, in a different historical moment or place. This is enough to confute any generalization regarding the colour of the crows. Most of the time, complete enumeration results to be impossible and generalization becomes a pure illusion. But most of the time doesn't mean always. We can, for example, admit that all books of logic, published during the 20th century, contain at least one formula, and we can evaluate the truth of this generalization. The set of logic books, unlike the one of crows, is a closed system, composed by members that are enumerable with certainty. Another paradox, created by Nelson Goodman,[74] is the 'grue and bleen' emeralds one. The uncertainty of all possible emeralds induces to adopt hybrid labels, because conceptual categories are always partial. Waiting a green emerald, because experience shows that emeralds are green, does not mean removing the possibility of the existence of a blue emerald. 'Grue' and 'bleen' concepts reveal generalizations that are valid, like any other, to describe the things of the world, but they are paradoxical because disrupt the same generalization.

Several antinomies exist also in causal induction. The paradox of donkey, created by Buridan,[75] for example, is very well known. The donkey is allowed to die when it is paralyzed by the doubt to choose between identical foods placed to its left and to its right. According to a deterministic perspective, the donkey's actions are caused by a binding condition. But, in a conflict situation, a well-defined cause doesn't exist. Identical causes cancel each other in not activating the animal's behaviour. Is the same situation valid also for humans? If we place a man between two perfectly identical tables filled with identical food, what would be his behaviour? Would he die for its uncertainty? This is, however, a remote possibility. Man seems to be free; he seems not to be bound to particular causal determinisms. However the explanation is not so simple. In fact, in a similar situation, we can consider our personal preference of the environmental side or the effect produced by a particular brightness, and so on. These are possible

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[73] Hempel, "Studies in the Logic of Confirmation I," "Studies in the Logic of Confirmation II," *Aspects of Scientific Explanation.*

[74] Nelson Goodman, *Fact, Fiction, and Forecast* (Cambridge: Harvard University Press, 1955).

[75] Clark, *Paradoxes from A to Z.*

subjective causes, but still deterministic. So, determinism, as an explication of causality, is not decidable.

The predictability of future events is another idea of causal induction. If the relation between cause and effect is known, then the prevision of future would be a certainty. Also this induction falls into a paradox, the prediction paradox.[76] In fact, if we can predict what will happen in future, starting from well-known causes, then we can also be able to change the situation. But, in this way the same starting predictions would be falsified, because the future is changed.

A reality that flees knowledge is not objectively and certainly knowable. Each human experience is an interactive dimension; it is a cumulative interference of reciprocity. Reality cannot be studied with detachment. Instead, each human experience is doubtful and unpredictable. Thus, reality cannot be completely generalized. It doesn't take clear forms; it is always potential; it is full of extraordinary possibilities that leave the mind in uncertainty and openness. Each observation and each induction are only open and hypothetic.

The open hypotheses facilitate logical investigation more than others. The hypothesis is a supposition based on the available data and proposed by the researcher to be verified or falsified. The hypothesis becomes an essential part of the inductive procedure, because it shows its possible conclusions in assuming general formulations that we must prove. When the hypothesis is verified, it becomes a momentary certainty. Thus, the infinity of inductive knowledge is confined to shared and validated principles that are never certain, but always falsifiable when the new data weaken their logical strength.

5. Pure Inductivism

Some scholars[77] rediscovered free inductive logic. It is not a slave of deduction. Instead, it is based both on unlimited investigation and on intuition. The pure inductive method is focalized on the continuous research of new problems, that is a boundless and never conclusive logical system. The revaluation of investigative thought, opposed to the static truth, highlights the inexhaustible creation of problems.

Beside the mentioned inductive modalities, this logic includes abstraction and abduction procedures.[78] Even abstraction is an induction, an essential induction. This is more a philosophical than a logical modality. It is centralized on the

[76] Clark, *Paradoxes from A to Z*.

[77] Nelson Goodman, *The Structure of Appearance* (Cambridge: Harvard University Press, 1951), *Fact, Fiction, and Forecast*, *Ways of Worldmaking* (Indianapolis: Hackett, 1978), Paul K. Feyerabend, *Against Method. Outline of an Anarchistic Theory of Knowledge* (London: NLB, 1975).

[78] Carlo Cellucci, *Le ragioni della logica* (Rome-Bari: Laterza, 1998).

analysis of the basic aspects of a particular nature.[79] Abstractive induction is the capacity of reason to exclude the superficial aspects of a problem and to focalize itself on the prior resolution aspects. This is a discrimination ability, which assists any logical course with sagacity and thoroughness. Instead, abduction regards the hypothetical inferences. The hypotheses formulation explains logical problems and it characterizes rational mind that are turned to discovery and investigation.

So structured, pure inductive logic follows an uninterrupted knowledge expansion. In fact, Carlo Cellucci[80] says, pure inductive logic is:

1) Augmentative. It evolves without stopping in introducing new and progressive adjustments. It moves from the problem to not decisive hypotheses.
2) Elastic in the formulation of hypotheses. This plasticity shows the absence of the rigidity of reason.
3) Modular[81] in the contextual application of the logical domains that are able to resolve different problems. The hypothesis is part of a continuous flux of changes.

During modern times, Francis Bacon[82] represents one of the most important exponents of inductive logic. He exalts induction, because he states that only it creates solid knowledge bases, which are empirically and scientifically founded. Any scientific idea is obtained through experience. The inductive reason processes data with order, without the arbitrary abstractions of the weak conclusions; the problems are resolved and compared to their real implications. The mind that starts from empirical data is free of beliefs and of preconceptions and so it can arrive at amazing discoveries. Thanks to the use of some exclusion tables and subsidiary instances that aim to exalt the phenomenal extremisms and excesses, the mind investigates reality and it selects from the available data the most significant ones. Bacon's induction proceeds gradually from data to hypotheses, to resolutions and to knowledge growth. This course makes a non-enumerative meaning but a selective one. Induction becomes purely definitional. Bacon's thesis introduces some reflections about the limits of pure inductive logic. The research falls into purely qualitative evaluations. This defect causes the overcoming of

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[79] Jacques Maritain, *Logica minore. Elementi di filosofia (II)* (Milan: Massimo, 1990). Juan J. Sanguineti, *Logica Gnoseologia* (Rome: Urbaniana University Press, 1983), *Logica filosofica* (Florence: Le Monnier, 1987).

[80] Cellucci, *Le ragioni della logica*.

[81] Types of modularity: 1) cooperation (compatibility between modules); 2) pre-emption (introduction of innovative modules); 3) negotiation (compromises between modules) (Cellucci, *Le ragioni della logica*).

[82] Francis Bacon, *Nuovo Organo* (Milan: Rusconi, 1998).

Bacon's logic in a historical period, between the 16th and the 17th centuries, signed by the birth of Galilean deductive method.[83] The need to give substance to knowledge deprives Bacon's method of real quantitative implications, so it becomes weak and uncertain in its conclusions. Pure inductive method results are poor and doubtful.

Instead, James Stuart Mill[84] pays attention to conceptual elaboration of some causal inductive methods and he prefers to study the differences, concordances and connections that occur between causes and effects. Thanks to Mill, pure inductive logic further shows its limits of formalization, because it becomes purely possible.[85] But, if this feature may not be a real constraint, then Mill's methods are surely weaker when they are applied to complex reality as, for example, social or psychical ones. Above all, this inductive procedure shows several difficulties in understanding the relevant causes[86] and it reproduces a sort of logic linearity that moves from problem to possible resolutions without deviations. This is an ingenuous logic because it doesn't represent the multi-linearity and the circularity of causal connections.

When inductive research becomes boundless, logical inexhaustibility doesn't have impositions and it dissipates knowledge in a confusing vanishing. Human knowledge must not encounter arrests and condensations; it must not languish in rigid stages, but it must lose itself in an uncurbed, relativistic and steamy growth. Any acquired knowledge becomes partial and so it assumes the characteristics of a despotic illusion, when it is considered very reliable. The truth is always adaptable to contingences and it is never absolute. The research shows the meaning of an investigative and indubitable instability. The infinity in progress dominates the logical outlook. Knowledge flows toward inexhaustibility and hypothetical method plays a central role on it. An unstoppable investigation process takes form. Each hypothesis becomes part of an unlimited hierarchy and of a continuity that, idealistically, starts from a problem and covers all human history.[87] From a relativistic point of view, knowledge becomes a constructive force that forms, through induction, some reality maps and this facilitates the creation of a purely subjective world.[88]

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83 Alfred N. Whitehead, *Science and the Modern World* (New York: Free Press, 1997).

84 James Stuart Mill, *A System of Logic: Ratiocinative and Inductive* (Ithaca: Cornell University Library, 2002).

85 Irving M. Copi, Carl Cohen, *Introduction to Logic* (Upper Saddle River: Prentice Hall, 2008).

86 Copi, Cohen, *Introduction to Logic.*

87 $Ip_1 \to Ip_2 \to Ip_3 \to Ip_4 \to \ldots \to Ip_n$. With $n = \infty$.

88 Goodman, *The Structure of Appearance*, *Fact, Fiction, and Forecast*, *Ways of Worldmaking.*

In *Pure Inductivism*, the hypothetical concatenation is infinite. The first hypothesis (*Ip1*) is not true if the second (*Ip2*) is false, and this hypothesis is not true if the third (*Ip3*) is false, and so on in an infinite regression. Hypothetical concatenation has no explicative value because everything flows in a linearity of conditionals. In this wild hypothetical method, no hypothesis is true if the conclusive hypothesis of the chain is false. However, this final hypothesis coincides with the infinity, which represents the necessary but not sufficient condition of all hypothetical inductive concatenation.[89] However, if we pose the true limit of the chain in a hypothetical infinite condition, then we establish a sort of paradoxical demonstrative self-reference that is clearly opposed to the open idea, typical of this logical model. If the infinity confirms the infinite hypotheses, then the circularity is evident and the relativism of knowledge becomes dominant.

Paul Feyerabend[90] expresses the peak of this relativism. In his ruthless attack against philosophy of science, considered useless or even parasitic, Feyerabend believes in a science that must enjoy of an absolute and anarchist freedom. Science should not have constraints, not even an illusory authority like reason. Feyerabend underlines that, at times of maximum scientific growth, figures such as Galileo came to their discoveries taking advantage on non-scientific capacities such as fantasy, cunning, rhetoric and propaganda. Without the silence of reason in the most important circumstances, science would grow less. Feyerabend concludes that inventiveness and creativity should not be inhibited.

5.1. The Realistic limits of Pure Inductivism and the Contained Logic

Pure inductive logic exalts the infinity of research and of human knowledge. This idea seems persuasive: it causes openness and freedom. Above all, it represents an important reaction to closures and contradictions of deductive logic. But it presents several limits.

The infinity misses the cumulative aspect of human knowledge. A not conclusive infinity does not give real knowledge. It becomes a confusing and vain research. Also the several promoters of pure inductive logic often remark the importance of the acquired knowledge and they underline that rational investigation must always adapt its courses to it.[91] Any acquired knowledge, however, presupposes a basilar and augmentative quiescence. The infinity is, thus,

[89] Peter D. Klein, "Human Knowledge and the Infinite Regress of Reasons." *Philosophical Studies* 134, 1 (2007): 1-17.

[90] Feyerabend, *Against Method*.

[91] Cellucci, *Le ragioni della logica*.

in the evolution of knowledge, in the conservation of some traits, in the change of other traits and in the general mutability. Each scientific progress is a profitable union of advances, arrests, involutions and unstoppable cognitive movements. Watching the history of human thought, we can discover that several theories persist for centuries, although changed or reduced. At the same time, we can discover that the other theories were buried by progress. Nevertheless, when there was a new discovery, a common knowledge base was present. Nothing of new arises without that something old is first widely accepted.

Some certainties seem to exist, but they are not useful to adopt an axiomatic logic. The idea of infinity must not deny the certainties, but it simply have to keep them in doubt because their future falsification is always possible.[92] However, during their existence, these certainties regulate any investigation and any formalization, thus they hold their same possible confutation. Idealistically, these certainties could always be valid because, like any other knowledge, they enjoy of infinity, which logically maintains them in doubt. Any certainty can be truthful, useful, fascinating and all comprehensive, but never immovable.

Pure inductive method considers hypotheses as methodological foundations of investigations, but it is a controversial method. Each question and each problem are reduced, in fact, to an only possible resolvable condition, so that referring to infinity presupposes a sort of intrinsic irresolution of the natural dilemmas. Instead, the hypotheses can capture important knowledge, established and accepted in their truthfulness, which persist over time in signing a continuative lull that leans towards the logical infinity, with the possibility to have an infinite lull. Resolvability becomes a journey of discovery where the formulated hypotheses can be provisory accepted or imperatively refuted or, sometimes, indefinitely preserved over the time. When obtained resolvability is high and effective, nothing is against the possibility to perpetuate the derived knowledge. But the primary hypothetical nature remains unchanged. Everything can be reviewed and refuted, but everything may remain unaltered. The hypotheses are the results of an active research. They are creative products that increase the growth of knowledge, maintain a link with the previous learning and impose themselves as evolutionary and non-invertible steps. So, we cannot say that any hypothesis resolves only specific problems, so as the exponents pure induction sustain.[93] The development of knowledge predisposes any discovery to more or less stable generalizations. The resolvability of a problem requires other similar problems. So, if knowledge is partial and incomplete in any specific moment, then these

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[92] Karl R. Popper, *The Logic of Scientific Discovery* (London: Routledge, 2002).

[93] Cellucci, *Le ragioni della logica*.

partiality and incompleteness decrease during the growth of knowledge. In an indefinite time, knowledge will become complete.

Thus, inductive logic is primarily an open mental attitude with a particular preference for what is inexhaustible, but it is also a conviction that nothing limits knowledge, not even the same infinity that takes on all axiomatization traits, when it is intended as a necessity. Therefore, as well as *Strong* and *Weak Deductivism*, *Pure Inductivism* is a closed logic.

5.2. The Foundations of Contained Logic

The idea of infinity does not escape, as the other logical ideas, from the power of paradox. If, for example, we compare the unlimited sequence of all natural numbers (*1, 2, 3, 4, 5,…, n*) with the sequence of the even numbers (*2, 4, 6, 8, 10,…, n*), then we can think that an infinite set corresponds to another infinite set that seems to be exactly its half. We can think that the infinite set of all natural numbers is more infinite than the infinite set of all even numbers. We can introduce a further idea of differentiation between infinity and numerousness of a sequence: while several numerical sequences can be equally infinite, some of these may be more numerous. The idea of infinity is extended by another specific term. Galileo solves this paradox.[94] In fact, he couples the sequence of double numbers (*2, 4, 6, 8, 10,…, 2n*) with the infinite sequence of natural numbers (*1, 2, 3, 4, 5,…, n*), in knowing that a double number is always an even number. Although we can consider the first sequence more numerous than the second sequence, every natural number can be paired one-to-one with a specific even number. Georg Cantor[95] creates a contemporary version of Galileo's paradox. Cantor's antinomy refers to the idea that each set is always strictly smaller than its power set,[96] which represents the class of all its sub-sets. Also the infinite set is small if it is compared with its power set.

The antinomies of nfinity date back to oldest times, when their narrative aspect was predominant. One of the most famous of them is the paradox of Achilles and tortoise described by Zeno of Elea.[97] Achilles decides to challenge a tortoise to a race but, being much faster, gives it an advantage. While he starts from the point p_1, the tortoise starts from the point p_2. But when Achilles reaches the point p_2, the tortoise is already at the point p_3, and when Achilles reaches the point p_3, the

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[94] Clark, *Paradoxes from A to Z*, Luminet, Lachiéze-Rey, *De l'infini…*

[95] Georg Cantor, *La formazione della Teoria degli Insiemi* (Florence: Sansoni, 2002), *Contributions to the Founding of the Theory of Transfinite Numbers* (New York: Dover, 2010).

[96] *X* is a set *{1,2,3}*. *P(x)* is the power-set of *X*. *P(x)* contains the empty set *{∅}* and the sets *{1}*, *{2}*, *{3}*, *{1,2}*, *{2,3}*, *{1,3}*, *{1,2,3}*.

[97] Clark, *Paradoxes from A to Z*.

tortoise is already at the point p_4, and so on ad infinitum. Whenever Achilles covers the new distance, the tortoise has already moved outdistancing him of a smaller and smaller stretch of road. The intervals that Achilles must cover become infinite. The stadium paradox of Zeno and the paradox of gods are very similar to that of Achilles and tortoise. In the first of these,[98] Achilles must run on the stadium track. But he cannot cross the finish line because he should cover a half of the remaining distance every time, but each section of the trail has its own half. Instead, in the paradox of gods described by Josè Benardete[99] a man wants to walk a mile. However, an infinite number of gods interferes with his path. When the man will reach a half of mile, one of these gods will intervene to put an obstacle. Another god will put, instead, a similar obstacle when the man will reach a quarter of mile, and another god will put it at an eight of mile, and so on ad infinitum. The man will be overwhelmed by the immensity of the obstacles, and so he cannot move from its starting point. But, finally, there will be no obstacles because the man does not move. He remains motionless simply because the gods' intentions will not be realized.

In conclusion, infinity is a persuasive idea and its direct experience is impossible. A limited condition, which can be existential, psychological or cognitive, makes man restless. A sense of unease reveals the utopias of perfection and certainty. An intuition vividly grows: something always escapes from these fatuous and illusory boundaries. Just intuition is, in fact, a basis of the idea of infinity. Finiteness is proper to human condition, but cosmos is not finite; it always extends its perimeters. But, when an idea derives from its original sources, it already forces and defines, in closing cosmos in boundaries. The idea of infinity, just like any other idea, occludes its own real content. In this way, the same idea distorts and betrays the indefinability and boundlessness of the original meaning. Infinity refers to what is always beyond human comprehension, so any attempt to think about it and to express it is a misrepresentation. This is the first fundamental contradiction of infinity. The idea of infinity forces to contain what is uncontainable second nature. It is not evidence and so it is inexpressible and it is indicative of an elusive quality. The second fundamental contradiction of infinity is its implicit lack of conformity with concrete reality. Experience is limited and limiting, although never conclusive.

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98 Clark, *Paradoxes from A to Z.*

99 Josè A. Benardete, *Infinity: An Essay in Metaphysics* (Oxford: Clarendon, 1964).

6. The Logical Limits of Scientific Knowledge

At this point, we can propose two questions: 1) How many are the logical limits of knowledge? 2) What are these limits? We can answer simply retracing our present work, but we must premise that a list of limits could never be complete. Furthermore, the Princeton Group (Piet Hut, David P. Ruelle, and Joseph F. Traub)[100] proposed an interesting list of the limits of knowledge: 1) curse of the exponential (some problems, such as chaos, can be solved in principle, but they are intolerably hard); 2) asking the wrong questions (there are structural limitations to the questions one may ask because some of these have no answers, they should not be asked, such as asking to specify the position and velocity of a quantum particle in a specific moment); 3) questions of questionable status (it is always unclear if a natural question that appears hard to answer corresponds to a fundamental limitation or to a bad problem); 4) emergent properties (the study and understanding of the higher levels of reality are very difficult); 5) limited access to data (in some areas of science, as the historical ones, the absence of sufficient data leads to severe limitations); 6) sample size of one (in sciences that deal with a single dimension, as cosmology, that studies the origins and structure of the universe, it is difficult to compare theories with observations); 7) technological limitations (some important limits affect the scientific practice, but the limits that can be overcome by new ideas are no real limits).

The logical form is common to all sciences, although it is different in its subjective manifestations, and so we can argue that each limit of scientific knowledge can be read in terms of logical limits, and vice versa. In fact, they are different terms that denote the same thing. According to our point of view, there are some extensive categories of limits that include the limits of Princeton's list:

1) Excess of logical rigor and categorical closures. The attempt to create complete and coherent knowledge systems collides with limitlessness of knowledge, so revealing the limits of demonstrative logic. In this category, we include 'asking the wrong questions,' because the fact that a question must not escape from demonstration criteria reveals its strong limitation, and 'questions of questionable status,' because several theories are not really reliable and complete.
2) Complementarity between objective and subjective worlds. The illusion of understanding reality, without considering human presence, collides with the complexity of cosmos, in revealing the limits of observational logic. In this

[100] Piet Hut, David P. Ruelle, and Joseph F. Traub, "Varieties of Limits to Scientific Knowledge," *Complexity* 5 (1998): 33-8.

category, we include 'limited access to data,' because all infinite data are not observable and collectible, 'technological limitations,' because the instrument needs continuous adjustments that follow the progress of knowledge, and 'sample size of one,' because in some sciences the comparison between theories and observations is very difficult.

3) Claims of hypothetical investigation. The intention to obtain certainty collides with the never decisive nature of the investigation, so revealing the limits of investigative logic. In this category, we include 'curse of the exponential' and 'emergent properties,' because some problems remain unsolvable at the current state of knowledge.
4) Dazzling inexhaustibility of hypothetical investigation. Infinity collides with its idea, so revealing the limits of the fatuous and ephemeral of scientific research. This category is not considered in Princeton's list and so it enhances it with a new element.

7. Some Considerations about the Changes and the Revolutions of Scientific Knowledge

In the course of our present work, we considered the main historical logical forms of scientific knowledge. Each of these forms, however, has shown some specific limits that have triggered revolutionary changes, when they became aware in scientific community. But, in talking about science revolution, we must compare our work with the ideas of Thomas Kuhn.[101] In fact, he also speaks about scientific revolution but he introduces some different concepts. Therefore, we believe that is necessary to develop a critical and integrative work. According to Kuhn, scientific knowledge oscillates between phases of stability and phases of change and revolution, which are announced by more or less deep crisis. During the phases of stability, or phases of normal science, scientific knowledge is structured around one or few basic rigid paradigms.

In a general meaning, the term 'paradigm' refers to the concept of "disciplinary matrix" that is the basic set of beliefs shared by a group of scientists.[102] According to the gestaltist perspective of Kuhn, the paradigm influences the perception of reality, and its subsequent interpretation. Moreover, because of different cultural and psychological conditions, in certain periods, these

[101] Thomas S. Kuhn, *The Structure of Scientific Revolutions (1962)* (Chicago: University of Chicago Press, 1970).

[102] Kuhn lists some beliefs that constitute the disciplinary matrix: 1) the symbolic generalizations; 2) the metaphysical paradigms or the metaphysical parts of paradigms, which are philosophical metaphors; 3) the scientific values; 4) the solutions of the typical problems.

paradigms highlight anomalies that, until then, had remained implicit or were considered not significant. When some authors explain these anomalies, they can trigger changes and revolutions. So, the paradigm changes drastically or it is replaced by another innovative paradigm. During the transition from the normal science to the revolutionary science, which will become normal in time, the paradigm is not rigid but changeable: this is a sort of pre-paradigmatic phase.

The common interest in scientific change allows a first and interesting correlation between the concept of 'paradigm,' which is fundamental in the study of Kuhn, and the concept of 'logical form,' which is instead basic in our present study. The two concepts are different by definition. While, in fact, the logical form is the rational style of scientific knowledge, the paradigm is instead its point of view on the world. Therefore, scientific knowledge constitutes their common nature. These two concepts, although different by definition, have also significant aspects of similarity in their common tendency to change, because all scientific knowledge is mutable. Indeed, we can argue that: 1) 'logical form' and 'paradigm' are both limited and ready to future processes of change; 2) these limits remain implicit until the scientific community take aware of them; 3) while the logical limits emerge as contradictions and inconsistencies, paradigmatic limits emerge as anomalies; 4) the change may eventually lead to a real scientific revolution that will be logical or paradigmatic. The paradigm and logical form are strongly linked, because their nature is common and their evolutions are similar. In fact, we think that scientific knowledge is logically structured around its paradigms; it processes the theoretical and methodological implications of its beliefs through a specific rational style. The same Kuhn, who relegates logic to a not significant component of paradigmatic change processing, implicitly identifies the logical form of each paradigm. In fact, in addition to the general definition of the term, he presents a more specific definition that, however, not too subtly conceals its logical sense. According to Kuhn, in starting from the usual problem, the scientist learns to see different situations as similar to each other, as subjects to the application of the same law or draft law. This learning is not verbal, but practical, because it resulted from the exercise. The scientist learns to recognize the similarity or dissimilarity between different examples and exercises. This definition, however, can be easily presented in logical terms. In fact, similarity relations are at the base of both the applications of deductive theories to multiply concrete examples and inductive generalizations of the analogy type: similar examples for the same theory in the first case; similar examples for the same generalization in the second case. We can therefore argue that each paradigm binds to a specific logical form that is deductive or inductive.

Given the link between the logical form and paradigm, we can also better explain the relations between the logical and the paradigmatic limits. In fact, no change is possible without a limit to exceed. The logical limits manifest themselves by paradoxes and structural antinomies, the paradigmatic limits by functional faults. In this regard, however, the historical study of science underlines an interesting and explaining characteristic: the emerging of the logical and paradigmatic limits, as well as their subsequent processes of change, may not always coincide and happen separately in different periods. So there are three possible conditions of change: 1) Only the logical limit emerges that starts a changing process or a revolution of the corresponding logical form; 2) Only the paradigmatic limit emerges that starts a changing process or a revolution of the corresponding paradigm; 3) Both of the limits emerge, at the same time or in different phases, which start a global change of the logical forms and their correspondents paradigms.

We can mention, as historical example of the first condition of change, the development of the third phase of Cognitivism, the Emergentism phase,[103] that took place between the eighties and the nineties of the last century. During these years, the American cognitivistic psychologists take awareness of the logical limits of their discipline, at that time divided into two approaches: 1) Computationalism, born in the first phase of Cognitivism, intends the mind as an highly symbolic cognitive dimension; 2) Connectionism, born in the second phase of the Cognitivism, on the other hand, intends the mind as a sub-symbolic neuronal dimension. According to Emergentism, Computationalism is too abstract while Connectionism is empty of any abstraction. The exponents of the rising Emergentism[104] want to integrate the two models into a single causal model that is, at the same time, top-down (cognition, as macroscopic dimension of the mind, causes neurological effects) and bottom-up (neurology, as microscopic dimension of the mind, causes cognitive effects). Mutual causing constraints are impressed between these two levels.[105] So the neuronal level is meant as a basic level from which it differs a higher level, with its own distinctive characteristics, which is

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[103] The notion of emergence implies the notion of novelty.

[104] The rise of this new phase is due to skills theorists. They study a type of learning not new but very complex, which is difficult to explain by the traditional models. The motor learning implies, in fact, very articulate behaviours that are hierarchical, non-linear and non-sequential (Karl S. Lashley, "The problem of serial order in behaviour," in *Cerebral Mechanisms in Behavior*, ed. Lloyd A. Jeffress (New York: Wiley, 1951), 112-146. There are in fact many variables that cannot be controlled (Nikolai A. Bernstein, *The Coordination and Regulation of Movements* (Oxford, New York: Pergamon Press, 1967)).

[105] Bechtel, *Philosophy of Science*.

the symbolic level of pure cognition. It places a bridge between biology and the psyche. The hierarchy represents an emersion, from the basic levels, of other levels more and more complex. The new Emergentism is a revolution in the logic of Cognitive Psychology, but not in its paradigms. The point of view on the mind is not changed: the mind maintains its psychic centrality, and Emergentism is not totally a new theory, but it is a summary of the integration between Computationalism and Connectionism. However, the basilar logical form is changed significantly. While Computationalism and Connectionism highlight a logical form of Weak Deductivism (empiricism serving theory), Emergentism highlights a logical form of Pure Inductivism (empiricism as an axiom of inexhaustibility). In the Emergentist phase, in fact, takes place the prospective of a definition of hierarchical horizons anchored to empirical and organic data that could be infinitary from an intuitive point of view.

As historical example of the second condition of change, we can mention the transition from Euclidean Geometry to non-Euclidean Geometries. Carl Friedrich Gauss's school is at the origin of this paradigmatic revolution.[106] He recognizes the impossibility to prove Euclid's fifth postulate[107] and he convinces himself about the legitimacy of to build up a coherent geometric system based on its own negation. Gauss's new Geometry reflects the proprieties of the space that are contradictory only in appearance. Janos Bolyai[108] and Nikolai Lobachevski[109] formalize the first real model of non-Euclidean Geometry. They proposed, similarly, a hyperbolic geometry. This geometry is based on the hypothesis of the acute angle of Giovanni Girolamo Saccheri[110] or Johann Heinrich Lambert, and establishes that for a point outside a straight line, it is possible to conduct only two parallel lines. In the second half of the nineteenth century, Bernhard Riemann postulated a second form of elliptic geometry.[111] This geometry, based on Saccheri's hypothesis of the acute angle, establishes the non-existence of the

[106] Harold S.M. Coxeter, *Non-Euclidean Geometry* (Whashington, D.C.: Mathematical Association of America, 1998).

[107] The Fifth Postulate, which is unintuitive, establishes that, given a straight line and a point external to it, one and only one other straight line, that is parallel to it, passes through that point.

[108] Janos Bolyai, *The Science Absolute of Space; Independent of the Truth or Falsity of Euclid's Axiom XI (which can never be decided a priori)* (University of Michigan: University Library, 2005).

[109] Nikolai I. Lobachevski, *Geometrical Research on the Theory of Parallels*, trans. George B. Halsted (New York: Cosimo, Inc. 2007).

[110] Giovanni Girolamo Saccheri, "Euclid Freed From Every Flaw (excerpts)," in *A Source Book in Mathematics*, ed. David E. Smith (New York: Dover Publications, 1959), 351-359.

[111] Coxeter, *Non-Euclidean Geometry.*

same parallel lines. Compared to Euclidean Geometry, non-Euclidean Geometries represent a change of paradigm, because the axioms change deeply, but the logical form of Strong Deductivism does not change.

As example of the third scientific change, we can also mention the rise of Darwinian Evolutionism that represents, in the nineteenth century, both a logical and paradigmatic revolution of science. Evolutionism defines the world as a set of constantly augmentative phenomena.[112] In fact, the concept of continuity represents the base of the new paradigm. Animal species constantly evolve to biological and mental forms always more functional and adaptive. The implicit inexhaustibility of the evolutionistic paradigm hides his logical inductive form, but science till the nineteenth century was not at all inductive. The revolution triggered by Evolutionism is therefore both logical and paradigmatic. This revolution proposes a sense of inexhaustibility that sensitizes the mind to philosophical Relativism. Also the rise of the Quantum Physics represents an example of this third condition of scientific revolution. The new paradigm introduced transforms, as a matter of fact, the idea of the sub-atomic world: Quantum Physics removes the previous distinction between particles and waves. A quantum system has the typical characteristics of the waves, but when it is measured, or even observed, takes on the characteristics of a set of particles (quanta). The new logical form abandons the old deductivisms, still dominant in Einstein's physics, to embrace a probabilistic perspective that is, instead, very indefinite and inductivist.

We present a final important difference between logical and paradigmatic revolutions of science. The paradigm, in fact, is a set of beliefs that is conditioned by historical, cultural and psychological contingencies. The logical form, instead, is a pure rational structure, less influenced by such contingencies. The history of science confirms this idea: while the paradigmatic changes and revolutions are the effect of some alternative tensions and tendencies that are increasingly evident in some historical period, the logical changes are instead the effect of the intuitions of some brilliant scholars. In fact, thinking back to the logical revolution of Gödel, we can see that it causes the collapse of Formalism in a time when it was strong and vigorous. Moreover, Gödel had no intention of demolishing the finitist illusion of Formalism. Historically, Formalism was not ready to die.

The work of Thomas Kuhn is a very important work, because of its insistence on the role of paradigms, but it is also objectionable, because it

[112] Pavel A. Florenskij, "Su un presupposto della concezione del mondo," in *Pavel A. Florenskij. Il simbolo e la forma*, eds. Natalino Valentini and Alexandre Gorelov (Turin: Bollati Borighieri, 2007), 13-24.

underestimates the role of reason in the development and the progress of science. Similarly to us, Imre Lakatos[113] recognizes the centrality of reason in each scientific revolution. According to him, if what he calls "research programme" of science is progressive, then reason supports knowledge and it solves and integrates the paradigmatic anomalies. If, instead, this programme is degenerative, then reason facilitates the paradigmatic change or the revolution.

We finally mention another important feature of the logical revolution: the immediate consequence of every great discovery. In moments of great discoveries, a sense of openness and innovation pervades science. But soon this 'spirit' becomes less intense, when a new and dominant logical form becomes more stable. When a logical form is stable it is also closed, but when the revolution is coming it can become open. A typical example is the birth of paraconsistent logics that we will analyze in the paragraph below, also for other reasons.

8. The Limits of Thought in the Innovative Tendencies of Logic

We can now consider some contemporary logical tendencies that constitute good examples of scientific changes and revolutions. We can take the example of the new paraconsistent logics. These tendencies also study the limits of thought, but with the intention to confirm their theoretical and methodological positions, in overshadowing the specific importance of the theme. Paraconsistent logics consider the contradictions as opportunities to extend the paradigm of logics. Some scholars[114] believe that several contradictions of thought are true. Thought uses its limits to access to alternative but true logical worlds, where the paradigms of classical logics collapse. More specifically, this logic abolishes the law of non-contradiction, so that everything can be true and false at the same time. In this way, paraconsistent logics aim to Logical Relativism, where it is possible to say everything and the opposite of everything, i.e. nothing. Paraconsistent logics are specific scientific knowledge, and therefore they take a specific logical form that is of Pure Inductivism.

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[113] Imre Lakatos and Alan Musgrave, eds. *Criticism and the Growth of Knowledge* (Cambridge: Cambridge University Press, 1970).

[114] Graham Priest, Richard Routley and Jean Norman, eds. *Paraconsistent Logic: Essays on the Inconsistent* (München: Philosophia Verlag, 1989), Graham Priest, *Beyond the Limits of Thought* (Cambridge: Cambridge University Press, 1995), "Paraconsistent Logic," in *Handbook of Philosophical Logic*, eds. Dov M. Gabbay and Franz Guenthner (Dordrecht: Kluwer Academic Publishers, 2002), 287-393.

Graham Priest[115] explains the legitimacy of paraconsistent logic in considering the limits of thought. In fact, these limits are occasions of illogicality, where thought is free from its rational constraints, so being able to create new and stronger logical systems. However, Priest unwittingly confuses the logical plan with the paradigmatic one, but he implicitly remembers to us the inseparability of the two plans. In fact, he defines four specific limits that we can present in logical and paradigmatic terms: 1) Limits of expressible. The features of the world transcend the ability of language to express them. Each point of view on the world is so limited. But in saying what those features are, we are liable to say the unsayable, and this is an evident logical contradiction. 2) Limits of iterable. Some operations are applied over and over again as far as possible. A representation of the world is constantly being proposed. The paradox of the mathematical infinity is typical: though there be no greater than the infinite, but there be a greater. But the paradoxes of the infinity are clearly logic. 3) Limits of conception. There are things beyond conception. Each point of view on the world may represent only a few things. But it is difficult to do so without conceiving them in some sense. Hence the logical contradiction at the limit of conception. 4) Limits of cognition. Several relationships arise between agents and the world that they cognise, between thought and the states of the world. However, at the same time, several limits arise as anomalies between representations and the things observed.

9. Conclusions

We think that a good closure of our argumentations has to say something about the possibility of solving and exceeding the limits of scientific knowledge. We believe that these limits provide an essential part of this knowledge. Some philosophers think, as happened in the case of David Hilbert and his school, that science is based on the idea of logical closure. Other philosophers think instead that science is based on the idea of perfection of its methods (as happened in the case of the members of the Vienna Circle). Finally, in the works of other philosophers, e.g. Paul Feyerabend, science loses itself in indefinable and relativistic perspectives. But every time, some brilliant scholar reveals the illusory that is hidden in these perspectives. In this regard, we can remember the works of Kurt Gödel, alan Turing, Werner Heisenberg, Bertrand Russell, and so on. In such works, scientific knowledge shows its real face. It shows that it cannot be closed, because it is open; that it cannot be completely open, because it contains; that it cannot be certain, because it is hypothetic.

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[115] Priest, *Beyond the Limits of Thought.*

At this point, we can essentially define science as a form of knowledge that is characterized by opening, possibility and containment. However, an essential definition of scientific knowledge requires an integrated definition of its characteristics. In this way, the risk of contradictions and paradoxes is avoided, as first of all the self-reference of terms: 1) opening is, at the same time, contained and possible; 2) containment is, at the same time, opened and possible; 3) possibility is, at the same time, contained and opened. Revolutions and stasis, reactions and innovations, creations and rationalizations are aspects of a wonderful human experience: science, whose weakening may cause limitations and contradictions.