QUANTUM VS. CLASSICAL LOGIC: THE REVISIONIST APPROACH[1]

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ABSTRACT: Quantum logic can be understood in two ways: as a study of the algebraic structures that appear in the context of the Hilbert space formalism of quantum mechanics; or as representing a non-classical logic in conflict with classical logic. My aim in this paper is to analyze the possibility to sustain, at least in principle, a revisionist approach to quantum logic, i.e. a position according to which quantum logic is 'the real logic' which should be adopted instead of classical logic.

KEYWORDS: quantum logic, quantum mechanics, logical constants, Hilary Putnam

Quantum logic can be understood in two ways: as a study of the algebraic structures that appear in the context of the Hilbert space formalism of quantum mechanics; or as representing a non-classical logic in conflict with classical logic. Within the second view on quantum logic we can distinguish between a preservationist approach, which accepts that quantum mechanics has a logic of its own, but consider that this does not force us to accept a logical revolution because we can understand this language of state attribution in quantum mechanics as a fragment of a language whose logic is classical.[2] A second approach would be the revisionist one, according to which quantum logic is 'the real logic' which should be adopted instead of classical logic, because the latter contains logical laws that must be rejected in all domains.

My aim in this paper is to analyze the possibility to sustain, at least in principle, a revisionist position. I will not be preoccupied here with all the problems that quantum logic deals with, nor will I try to answer to the objections that were brought against adopting a revisionist approach. These are best dealt with when they are taken into consideration from a crystallized position and they will, therefore, not make the object of this paper.

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[2] See for example Bas van Fraassen, *Quantum Mechanics: An Empiricist View* (Clarendon Paperbacks, 1991), 128-135.

The starting point: Quine 1 or Quine 2?

We can distinguish between two types of logical revision:

1. *a weak sense:* we start from an analysis of the meaning of sentences to show that certain classical logic laws do not apply to the sentences of a certain type.[3]
2. *a strong sense*: we start from the empirical data and we argue that we must operate this kind of revision as a response to these data.

Quantum logic is a case of revising logic in a strong sense.

The possibility of revising logic in a strong sense is brought into attention for the first time by Quine. It follows as a consequence of adopting his holistic view on knowledge. According to this view,

> The totality of our so-called knowledge or beliefs, from the most casual matters of geography and history to the profoundest laws of atomic physics or even of pure mathematics and logic, is a man-made fabric which impinges on experience only along the edges. Or, to change the figure, total science is like a field of force whose boundary conditions are experience.[4]

When confronted with a recalcitrant experience, the revising of the system is taken into consideration. This revision can take place at different levels in the system, but usually

> the more fundamental a law is to our conceptual scheme, the less likely we are to choose it for revision. When some revision of our system of statements is called for, we prefer, other things being equal, a revision which disturb the system least.[5]

Since the revision of logic would cause the biggest disruption of the system, the quantum logician has to offer very powerful reasons for the proposed revision – reasons that would make us prefer this revision instead of other alternatives. For example, in the case of physics, as Reichenbach shows, if we want to maintain a Euclidian geometry, we must be willing to accept all sort of causal anomalies: mysterious forces, instantaneous actions at a distance, infinite reduplication, etc. In this context, instead of having to deal with all these anomalies, a revision of mathematics seems preferable.

Quine offers as an example of a case in which the revision of logic can be taken into consideration, the case of quantum mechanics.

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[3] The best known case of logical revision in a weak sense is intuitionistic logic.

[4] Willard Van Orman Quine, *From a Logical Point of View: 9 Logico-philosophical Essays* (Cambridge: Harvard University Press, 1961), 42.

[5] W.V. Quine, *Methods of Logic* (Cambridge: Harvard University Press, 1982), 2.

Later, Quine changes his mind about this topic and claims that we cannot have alternative logics to classical logic in the sense of logics that reject any of the logical truths as falsehoods.[6] He claims that denying a logical truth means changing the subject, because, if we do not accept a law of classical logic, then we do not attach to the logical constants that appear in this law the same meaning attached by the classical logician.[7]

But we can still sustain the need of a logical revision making abstraction of these issues, if we use the analogy with the case of Euclidian geometry.

Putnam and quantum logic

In his paper "Is Logic empirical?," Hilary Putnam[8] asks the following question: in the case of Euclidian geometry, has happened that 'truths' which were thought to be necessary to be rejected as falsehoods, why wouldn't it be also the case that some 'necessary truths' of logic be rejected?[9] Since Einstein proposed his General Theory of Relativity, the idea that Euclidian geometry represents the mathematical frame suited for formulating empirical laws that describe some concrete empirical phenomena was put aside. That determined some philosophers to say that if the General Theory of Relativity is correct, then some 'truths' thought to be necessary are rejected as false and thereby the whole class of 'necessary truths' is considered problematic. An example of such a 'necessary truth' would be that of the following Euclidian axiom: "the shortest distance between two points is a straight line." If we accept the General Theory of Relativity, we also accept that it is possible that the shortest distance between two points is not a straight line, but a geodesic (this happens, for example, in a strong gravitational field, as that of the Sun) and thereby we accept that there are empirical situations in which the Euclidian axiom is false. Pushing things further on this line we can ask: why wouldn't it be the case that some laws of logic are false? Those who accept a logical interpretation of quantum mechanics will say that there are such laws of classical logic that are false. They consider that the true conceptual revolution produced by the quantum mechanics is the revision of logic.

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[6] For example in W.V. Quine, *Philosophy of Logic, 2nd Edition* (Cambridge: Harvard University Press, 1986), 80-94.

[7] Alan Berger offers a detailed discussion of Quine's new position in Berger, "Quine on 'Alternative Logics' and Verdict Tables," *Journal of Philosophy* 77 (1980): 259-277.

[8] Hilary Putnam, "Is Logic Empirical?" in *Boston Studies in the Philosophy of Science*, vol. 5, eds. Robert S. Cohen and Marx W. Wartofsky (Dordrecht: D. Reidel, 1968), 216-241.

[9] Putnam, "Is Logic Empirical," 216.

According to Putnam,[10] the core of the logical interpretation of quantum mechanics is the following identity:

$$\frac{\text{geometry}}{\text{General Theory of Relativity}} = \frac{\text{logic}}{\text{Quantum Mechanics}}$$

Considered to be the most successful theory in the history of science because of its predictive power, quantum mechanics was even from the beginning a theory in search of an interpretation. When we try to interpret the mathematical formalism of this theory, we face some problems generated by the fact that, as it seems, we can not give an interpretation that does not violates one of the fundamental principles of classical physics, e.g. of causality, of energy conservation, etc. It seems that such an attempt to understand the world described by this theory requires the revision of our understanding of the nature of things (the objective nature of reality, its dependence on our perception, the nature of a complex system and its relation to its parts, etc.[11]), or even, according to some, the change of logic.

Quantum mechanics and its interpretation

The standard interpretation of quantum theory is the Copenhagen interpretation which has Niels Bohr, Werner Heisenberg, Max Born, etc., among its founders. This interpretation is based on the principle of uncertainty, the particle-wave duality, the probability interpretation of the wave function given by Born, the interpretation of the eigenvalues as measured values of the observables, and the principle of correspondence.

The uncertainty principle: the measurable physical properties of a quantum system are incompatible with each other, i.e. measuring one will affect the other. Therefore, no quantum state can generate simultaneous high probabilities concerning two observables, e.g. position and momentum. From Heisenberg's perspective, this type of inverse relationship shows that the mathematical representatives for observable quantities, the Hermitian operators, are non-commutative. The main idea here is that any measuring of a property of a system affects, inevitably, the system.

According to Heisenberg, in its first interpretation of the principle, the essential element of the quantum theory is the inevitability of a minimum interference in the system, the impossibility of not disturbing its state. Therefore,

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[10] Hilary Putnam, "How to Think Quantum-Logically", *in Logic and Probability in Quantum Mechanics*, ed. Patrick Suppes (Dordrecht: Reidel, 1975), 47.

[11] Lawrence Sklar, *Philosophy of Physics* (Oxford: Oxford University Press, 1992), 157.

the uncertainty appears as a limitation of our abilities to discern simultaneously the exact values of two conjugate properties of a system.

Bohr was not pleased with this interpretation and he insisted that the specification of a quantum state of a system represents a complete description of the system about which the quantum state was correctly predicated.

The probability interpretation of the wavefunction: Born interpreted the wavefunction as giving a probability. He was influenced by Einstein's suggestion that, for photons, the wave field acts as a weird certain type of 'ghost' field, which guides photons on tracks that can thereby be determined by the effects of interference of the waves. Thus, Born's interpretation is that the square of the amplitude of the wave function in a certain specific area of the configuration space is connected with the probability of finding the corresponding quantum particle in that area of the configuration space.

According to Born, the wave function represents the evolution of the state of our knowledge about a quantum system.

Interpreting the eigenvalues as measured values of the observables: the eigenvalues of operators correspond with the measured values of the observables for which the operators stand.

The principle of correspondence: everything that the quantum theory will predict about the quantum particles has to be consistent with macroscopic particles behaving in the Newtonian way. This requirement can be transformed in a prescription for calculating the commutators of the operators of the observables of quantum particles starting from the mathematical relations between the correspondent measurable properties of the classical theory.

Beside these principles, quantum mechanics, in its standard interpretation, has the following postulates:

1. (a) the state of a particle is represented by a vector in a Hilbert space.

   (b) the state of a quantum-mechanical system is completely described by the wavefunction ψ.

2. the observable quantities are represented by Hermitian operators that are consistent with the commutation relation between position and momentum $[x,p] = i\hbar$

3. a quantum system evolves according to Schrödinger's equation ($i\hbar\, d/dt\, |\psi(t)> = H\, |\psi(t)>$) as long as no measurement is made.

4. the result of the measurement of one property of a system, given the initial state of that system, can not be known with certainty in advance.

5. if a particle is in the state $|\psi>$, the measurement of the variable that corresponds to Ω will have as a result one of the eigenvalues ω with the probability that

> $P(\omega) \propto |\langle\omega|\psi\rangle|^2$, i.e. in case of a measurement, the state vector of a system collapses into an eigenvector of the measured observable operator.

Problems with the standard interpretation

As all the other interpretations of the quantum theory, the standard interpretation tries to avoid certain paradoxes that appear when the world of subatomic particles is taken into consideration. The problem with this interpretation is that it forces us to accept indeterminist laws, nonlocality, instrumentalism and subjectivism. Therefore, if we see physics as an aspiration to produce a true description of an objective reality, that is if we are scientific realists, we can only be unpleased with the Copenhagen interpretation. Among those unsatisfied with this interpretation was Einstein, who considered that the quantum-mechanical description is incomplete:

> If, in quantum mechanics, we consider the Ψ-function as (in principle) a complete description of a real physical situation we thereby imply the hypothesis of action-at-distance, an hypothesis which is hardly acceptable. If, on the other hand, we consider the Ψ-function as an incomplete description of a real physical situation, then it is hardly to be believed that, for this incomplete description, strict laws of temporal dependence hold.[12]

Barry Loewer identifies three problems that appear in connection with the Copenhagen interpretation:

- *it is vague*, because it does not say what type of interactions measurements are, and this omission is important because, according to this interpretation, the unmeasured systems evolve according to Schrödinger's law, and the measured ones according to the collapse postulate.
- *it is inconsistent*, because it makes assertions about the nature of the quantum-mechanical reality, but denies that one can know anything about that reality.
- *it is obscure*, because the relationship between the measurement and reality is obscure: what is so special about the measurements that they initiate the collapse?[13]

Another problem with this interpretation would be that it does not explain why the quantum states do not resemble at all the classical ones.

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[12] Albert Einstein, summary to "Quanten-Mechanik und Wirklichkeit," *Dialectica* 2 (1948): 324.

[13] Barry Loewer, "Copenhagen versus Bohmian Interpretations of Quantum Theory," *The British Journal for the Philosophy of Science* 49 (1998): 3.

The quantum logic interpretation

We can obtain alternative interpretations in two ways: either change the physical postulates of the quantum theory,[14] or change logic.

According to David Finkelstein, "one of the reasons that is so difficult to understand quantum mechanics is that our teachers fail to tell us it is illogical, violates the canons of classical logic."[15] For example, if we take into consideration a particle, we can say about that particle **(Er)** – the position of the particle is r, and **(Er')** – the momentum of the particle is r'; but we can not draw the conclusion that **(Er)(Er')(the position of the particle is r, and the momentum is r')**.[16] Thereby, the following equivalence, which is a theorem in classical logic, is rejected: **(∃x) Fx & (∃y) Gy ↔ (∃x) (∃y) Fx & Gy**. Starting from here, those who adopt a logical interpretation of quantum mechanics say about all the logical relations that hold between the empirical/physical states that they are an empirical matter and are not given a priori.

We can argue in several ways that, in the case of quantum mechanics, the underlying logic of the events is a non-classical logic:

> *(A)* one way would be that of identifying 'the logic' of the probabilistic theory with the algebraic structure of the set of events to which the probability is assigned. But the algebra of the events in quantum mechanics is not Boolean and therefore neither the logic can be Boolean.[17]

For example, in the case of the double-slit experiment,

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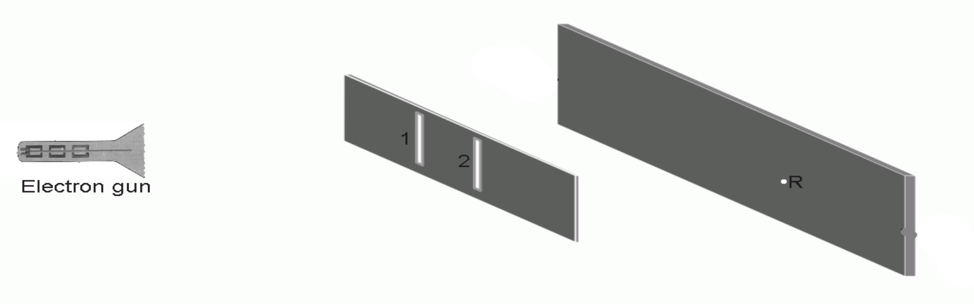
[14] The case of Bohm's interpretation, for example.

[15] David Finkelstein, "Matter, Space and Logic," in *Boston Studies in the Philosophy of Science*, vol. 5, 203.

[16] The uncertainty principle stops us from doing this.

[17] We can say about a given set L that it is a Boolean algebra, if the following laws are satisfied by it: :

- $x \cup y = y \cup x$, $x \cap y = y \cap x$ – the commutative law
- $x \cup (y \cup z) = (x \cup y) \cup z$ – the associative law
- $x \cup (y \cap x) = (x \cup y) \cap x$ – the absorption law
- $x \cup (y \cap z) = (x \cup y) \cap (x \cup z)$ – the distributive law
- $x \cup x = x \cap x = x$ – the idempotent law
- **the law of complementarity:** if there is a smallest element 0 and a largest element I, then for every element x there is an element x' that satisfies the following: $\mathbf{x \cup x' = \mathit{I}; x \cap x' = 0}$.



if $P(A_i, R)$ is the probability that an electron passes through i $(i = 1,2)$ and hits region **R** of our screen, and if the emission pattern is symmetrical, we have:

$$P(A_1) = P(A_2) = P(A_1 \vee A_2) / 2.$$

Then we can derive the following equation:

$$P(A_1 \vee A_2, R) = P(A_1 \vee A_2, R)/ P(A_1 \vee A_2)$$

$$= P(A_1, R \vee A_2, R) / P(A_1 \vee A_2)$$

$$= P(A_1, R)/ P(A_1 \vee A_2) + P(A_2, R)/ P(A_1 \vee A_2)$$

$$= P(A_1, R)/ 2\, P(A_1) + P(A_2, R)/ 2\, P(A_2)$$

$$= ½\, P(A_1, R) + ½\, P(A_2, R)$$[18]

The problem we confront with is that this equation does not hold in quantum mechanics.

> **(B)** another way would be to "just read the logic off from the Hilbert space H(S)."[19]

In quantum mechanics, the state of a physical system **S** is represented by a vector in a Hilbert space **H(S)**. An assertion about **S**, e.g. **m(S) = r** (the physical quantity m has the value **r** in system **S**) is coordinated with a subspace **S(p)** of **H(S)**, where **p** is **m(s) = r**. We can form complex propositions in this context, as following: $S(p \oplus q)$ = the span of **S(p)** and **S(q)**; $S(p \cap q)$ the intersection of **S(p)** and **S(q)**; $S(\perp p)$ = the ortocomplement of **S(p)**.

We can establish the following equivalence between the logical connectors **V, &, ˜** and the operations of reunion, intersection and complementary of the lattice formed by the propositions of the language of state attribution in quantum

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[18] See Putnam, "Is Logic Empirical," 223; Michael Gardner, "Two Deviant Logics for Quantum Theory: Bohr and Reichenbach," *British Journal for the Philosophy of Science* 23 (1972): 90.

[19] Putnam, "Is Logic Empirical," 222.

mechanics: V (disjunction) corresponds to ⊕, & (conjunction) corresponds to ∩, ˜ (negation) corresponds to ⊥. Now it is easy to understand why the distributive law does not hold in quantum mechanics: the lattice with whose operations the above logical operations are equivalent is non-distributive. Therefore in the resulted logic it can not appear the distributive law from classical logic.

I said above that the quantum logician has to offer strong reasons for adopting the revision he proposes. Putnam offers such reasons. If we accept this revision, we can offer a realist interpretation for quantum mechanics and we escape all the anomalies put forward by the other interpretations.

We can distinguish the following features of the quantum-logic interpretation of quantum mechanics:[20]

1. the measurement does not produce the observable measured and does not determine something that was not already the case. It is a physical interaction as any other.
2. the probability enters the quantum theory as it enters the classical physics.
3. the Hilbert spaces used in quantum mechanics are only mathematical representations of some logical spaces: there is an isomorphism between the lattice formed by the subspaces of a Hilbert space under the relation of 'the subspace of' and the lattice formed by the physical propositions about the quantum system under the relation of implication.[21]

We can see from the map traced above that we have two alternatives: either keep the classical logic and accept a paradoxical physics, or adopt a new logic and escape this way the paradoxes.

Prospects for a revisionist approach

A powerful challenge for the revisionist would be that of showing that the quantum logic is 'the true logic' and that we must abandon classical logic. So, he must answer to the preservationist, who argues that "the little language of state-attributions, whose 'inner logic' is not classical, is a fragment of a larger language, whose logic is classical."[22] The argument offered by the preservationist is very simple: the logic of the language in which we made this presentation of quantum logic is classical. We didn't have to abandon classical logic in order to understand it.

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[20] In Putnam's view. See his "How to Think Quantum-Logically."

[21] Putnam, "How to Think Quantum-Logically," 49-51.

[22] van Fraasen, *Quantum Mechanics,* 135.

To find a way out of this difficulty, we must turn our attention for a moment to philosophical logic, especially to the problem of logical constants, i.e. the logical vocabulary. How do we delimitate this vocabulary from the extra-logical one? We can distinguish between three answer strategies: (i) the logical vocabulary is simply specified by enumeration; (ii) we seek a criterion for constancy; (iii) by searching for an understanding of the goal of a logical theory and the way in which this goal has to be achieved.[23]

In the remaining of this paper I will present broadly the third strategy as it appears in Warmbrod's paper because, as I will show at the end, the alternative proposed by Warmbrod to the discussed strategies helps us very much in our project of establishing the possibility of adopting a new revisionist position.

I said that in the third strategy of response, *we search for an understanding of the goal of a logical theory and the way in which this goal has to be achieved.* Warmbrod distinguishes between two ways of understanding these goals:

> a) what it is aimed is the formal characterization of the logical consequence and the logical truth on the basis of some pre-theoretical intuitions about necessity, apriority and form. From this perspective the logical constants are those terms whose meaning is fixed in order to have a theory of logical consequence and logical truth that is in accordance with these pre-theoretical intuitions.[24]

The thing with this way of characterizing logical consequence and logical truth is that when we try to see the logical theories in light of the fact that they respect or not certain pre-theoretical intuitions, we stumble upon the following problem: these intuitions are very controversial:

> (i) in the case of necessity and a priori knowledge it is well known Quine's critique.
>
> (ii) in the case of the intuitions concerning the logical form, it is unclear if ordinary people have these sorts of intuitions.

As it can be seen, this strategy goes into conflict with a generally accepted idea, that the foundations of logic should be as safe and as less controversial as possible.

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[23] For a detailed discussion of these strategies, see Ken Warmbrod, "Logical Constants," *Mind* 108 (1999): 503-538.

[24] Tarski, for example, appeals to the intuitions about necessity in order to justify the definition given to the logical consequence. See also Gila Sher, who considers that the distinction between the logical and extra-logical terms is based on our pre-theoretical intuitions that logical consequence differ from the material ones by being necessary and formal.

b) what we are after is to find a conceptual framework suited for the project of the deductive systematization of scientific theories. This is the alternative proposed by Warmbrod to the above strategies. Warmbrod considers that a logical theory includes (i) a core theory meant to characterize the logical consequence and the logical truth in a way that avoids the appeal to intuitions and which provides a frame in which other more controversial theories can be formulated; (ii) an extended theory that consists of different theories formulated as extensions of the core theory. These extensions are intended to formalize our intuitions about necessity, aprioricity, etc.

1. *The core theory* – is built as a theory of the deductive systematization: what we are concerned with is the contribution of logic to the scientist's task of constructing and testing theories about the world. Warmbrod distinguishes the following things the scientist is interested in:

(i) to clarify the sentences made by the theory. This deductive systematization will help reach this objective as follows: once we have a definition of the logical consequence, the set of sentences of the theory can be clarified by choosing a set of axioms.

(ii) to communicate the theory to other scientists. Because the set of sentences of the theory is infinite, it can not be communicated by a list. If we have a logical theory, this problem is solved because the scientific theory can be presented by listing the axioms and indicating the assumed logical theory.

(iii) to allow the systematic testing of the theory. This task is realizable only if the notion of logical consequence is truth-preserving. For that we need to link the concept of logical consequence to the concept of truth. But since the concept of truth is a semantic one, a purely syntactic explanation of the consequence relation is out of the question and we must appeal to a theory of truth that specifies a truth condition for each sentence of the language. At this moment, to define the logical consequence, we can use Tarski's suggestion concerning the connection between the consequence relation and truth. We have the following definition: The logical consequence is a relation that holds when all the permitted assignments that make the premises true also make the conclusion true.

(iv) to allow a comparison of the theories. The logical theory suited for this task is a theory that solves as few of the theoretical controversies as possible.

The problem that remains is to show which are the logical constants. At this point we should take into account in our decision the scientist's intuitions about the theory's content (these are pre-theoretical identified). Another thing that we should take into account is the minimalist constraint which says that a logical theory suited for the purpose of providing a conceptual apparatus for the project of systematization should be as simple, as modest in assumptions and as flexible as possible.

What set of constants is adequate? First of all, we should count the veri-functional connectors. We need the negation '~', because every scientific theory postulates entities and claims that an entity satisfies or not certain conditions. Also, any theory will claim that an entity satisfies one out of two conditions, without specifying which, thus we need the disjunction '**v**.' If a logical theory contains '~' and '**v**,' it must also contain all the veri-functional consequences of them.

2. *The extended theory*. Beside this conceptual framework adequate to the deductive systematization of science, we might search for a theory that is richer than the core theory, but which remains neutral, in the sense that it doesn't presuppose anything about the type of entities that exist. Also, we might want to formalize not only theories about the world, but a particular mode of talking about the world.[25]

The reason why I made this long detour from our main subject should by now be transparent. We know that choosing the logical constants determine which propositions are logical truths. If we adopt Warmbrod's strategy of choosing these constants, we obtain a way of justifying the revisionist approach. Let's take just one case: in quantum mechanics, for any two propositions **a** and **b**, their disjunction (**a v b**) can be true without either **a** or **b** being true. Thus, when we choose the set of adequate constants for the task proposed by Warmbrod, we choose the constants of quantum logic, not of classical logic.

The preservationist can reply here that this is in perfect agreement with his position: quantum mechanics has a logic of its own, but when scientists are reasoning about the macroscopic world they are not using quantum mechanics, so quantum logic should be restricted to the quantum world. This is possible only because the subatomic world of quantum particles is very different from the macroscopic world, and our present quantum theory doesn't explain why the world of our everyday experiences looks the way it does. But, if we are optimistic about the chances of finding a unified theory, we have to be optimistic about the prospects of replacing the classical logic with something at least resembling quantum logic.

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[25] Warmbrod, "Logical Constants," 517-536.