

FREGE ON IDENTITY. THE TRANSITION FROM *BEGRIFFSSCHRIFT* TO *ÜBER SINN UND BEDEUTUNG*

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ABSTRACT: The goal of the paper is to offer an explanation why Frege has changed his *Begriffsschrift* account of identity to the one presented in *Über Sinn und Bedeutung*. The main claim of the paper is that in order to better understand Frege's motivation for the introduction of his distinction between sense and reference, which marks his change of views, one should place this change in its original setting, namely the broader framework of Frege's fundamental preoccupations with the foundations of arithmetic and logic. The Fregean thesis that mathematics is contentful, and its defense against *formalism* and *psychologism*, provides us an valuable interpretative key. Thus, Fregean senses are not just the mere outcome of some profound reflections on language, rather they play an important role in the articulation of Frege's program in the foundations of arithmetic

KEYWORDS: Fregean senses, informative identity, contentful mathematics

1. Introduction

Frege's account of identity is puzzling, and his views on this subject continue to occupy contemporary philosophical discussion. This paper aims to explain why and how Frege made the transition from his theory of identity proposed in *Begriffsschrift* (hereafter, *Bgs*)² to the one presented in *Über Sinn und Bedeutung* (hereafter, *SB*).³ Recently, a series of papers⁴ dedicated to this subject have

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² Gottlob Frege, *Begriffsschrift, eine der arithmetischen nachgebildete Formelsprache des reinen Denkens* (Halle: I. Nebert, 1879), translated in Gottlob Frege, *Conceptual Notations and Related Articles*, trans. and ed. Terrell Ward Bynum (Oxford: Oxford University Press, 1972), and selections in *The Frege Reader*, ed. Michael Beaney (Oxford: Blackwell, 1997).

³ Gottlob Frege, "Über Sinn und Bedeutung," *Zeitschrift für Philosophie und philosophische Kritik* 100 (1892): 25-50, translated as "On Sense and reference" in *Translations from the*

appeared in the *Canadian Journal of Philosophy*, but unfortunately none of them explains conclusively Frege's motivation for this change. After presenting some recent contributions to this debate, I will focus on what I think was Frege's motivation for changing his views on identity. The main claim of the paper is that, in order to better understand Frege's motivation for the introduction of his distinction between sense and reference, we should seriously consider its original setting, namely the broader framework of Frege's fundamental preoccupations with the foundations of arithmetic and logic. The 'standard interpretation' is basically the narrow interpretation which holds that in *SB* Frege criticizes and rejects the account of identity of *Bgs*. The standard interpretation considers Frege's change of view only within the framework of philosophy of language, and assesses his theory of meaning solely from this perspective. In contrast with this point of view, I advocate an interpretation which considers his views on identity in the wider context on mathematics and logic.

Mike Thau and Ben Caplan⁵ attacked the 'standard interpretation' and held that Frege never gave up his *Begriffsschrift* account of identity. I believe that their interpretation is mistaken, and I think that Richard Heck⁶ has refuted this position conclusively. My goal here is to show why Frege came up with a new view of identity, thus completing Heck's refutation of this attack on the standard interpretation. What is wrong with the standard view is not that it claims that Frege changed his position concerning identity, but its failure to consider the rationale underlying this change. Based on the traditional way in which one has commonly learned that philosophy of language and philosophy of mathematics are disconnected philosophical fields, our natural inclination is to judge things separately; this approach is mistaken, and we shall shortly see why.

A recent affirmation of Thau and Caplan's claim that in *SB* Frege did not reject his earlier view of identity in *Bgs* may be found in Bar-Elli.⁷ Basically, Bar-

Philosophical Writings of Gottlob Frege, eds. Peter Geach and Max Black (Oxford: Basil Blackwell, 1960), and in *The Frege Reader*.

⁴ Mike Thau, Ben Caplan, "What's Puzzling Gottlob Frege?" *Canadian Journal of Philosophy* 31, 2 (2001): 159-200; Richard G. Heck, "Frege on Identity and Identity-Statements: A Reply to Thau and Caplan", *Canadian Journal of Philosophy* 33, 1 (2003): 83-102; Gilead Bar-Elli, "Identity in Frege's *Begriffsschrift*: Where Both Thau-Caplan and Heck Are Wrong," *Canadian Journal of Philosophy* 36, 3 (2006): 335-370; Imogen Dickie, "Informative Identities in the *Begriffsschrift* and 'On Sense and Reference,'" *Canadian Journal of Philosophy* 38, 2 (2008): 269-288.

⁵ Thau, Caplan, "What's Puzzling."

⁶ Heck, "Frege on Identity and Identity-Statements."

⁷ Bar-Elli, "Identity in Frege's *Begriffsschrift*," 357.

Elli's main claim is that in *Bgs* "Frege distinguishes there between names (*Namen*) and signs (*Zeichen*). The distinction is not explicitly stated, but it is used almost consistently in section 8. (...) A sign, in *Bs*,⁸ just denotes its content; this exhausts its meaning. A name, in contrast, includes a mode of determination (*Bestimmungsweise*) of its content." Bar-Elli holds that we should distinguish a 'thin' semantics, in which signs refer directly to their contents, and a 'thick' semantics, in which names refer to their referents through a sense or a mode of determination. Thus, for Bar-Elli, the transition from *Bgs* to *SB* is the transition from the coexistence of a semantics of signs and a semantics of names in *Bgs* to the unified thick semantics of *SB*.

At least one issue is problematic here: the allegation that Frege distinguished between signs and names. Here, I deal only with the former point. Regarding the distinction between signs and names in *Bgs*, at least two points should be noted: First, in §1 Frege makes a distinction in the realm of signs between variables and constants. But what are names if not constants? So, at most, we can say that names are a subclass of signs. Second, in §8, Frege presents a geometrical example in which the apparently different points *A* and *B* are in fact one and the same, the difference between them consisting in the way in which they are determined. And it is true that in connection with these different 'modes of determination,' *A* and *B* are also called 'names,' but, at the very end of the section, Frege says explicitly:

Now let

├—(*A* ≡ *B*)

mean that the sign *A* and the sign *B* have the same conceptual content, so that we can everywhere put *B* for *A* and conversely.

Since Frege does not distinguish here or elsewhere between names and signs, the distinction between a thin and a thick semantics seems to be an unsustainable interpretation of Frege's semantics.⁹ Moreover, what Bar-Elli calls a 'thin semantics' is not part of the Fregean view of how signs/names refer. Frege says explicitly in *Bgs* §8 that "one point is determined in two ways: (1) immediately through intuition and (2) as a point *B* associated with the ray perpendicular to the diameter." Thus this so-called 'thin semantics' (basically, a Millian view of proper names) is seen as a case of a thick semantics: to be determined directly in intuition is, for Frege, just another way of being

⁸ Bar-Elli's shortcut for *Bgs*.

⁹ As we will see shortly, for Frege mathematical signs, including mathematical names, have content in the sense that they do refer to objects.

determined. Moreover, since the difference between the thin and thick semantics is based on the alleged difference between signs and names, it is necessary to understand why Frege held this in *Bgs* but dropped it later. No explanation is offered by Bar-Elli.

In another recent paper dedicated to this topic, Imogen Dickie¹⁰ explains the transition between Frege's account of the informativeness of identity statements from *Bgs* to *SB* in terms of a difference between two senses of 'informative.' Dickie holds that in *Bgs* Frege is concerned with 'evolutionary informativeness': the transition, from the fact that a subject may understand two co-referential names without knowing that they co-refer, to the situation when the subject finds that they do co-refer, constitutes an epistemic advance. Dickie holds that in *SB* Frege is concerned with 'rational informativeness': the substitution of co-referring expressions in a proof preserves truth, but may transform a logically self-evident chain of inferences into one which is not, or vice-versa. This distinction is subtle and interesting, yet Dickie doesn't offer much textual evidence to show that these different senses of informativeness are separately connected with *Bgs* and *SB*. There are three questions to be addressed here. First, why cannot that which is informative in the evolutionary sense also be so in the rational sense, and conversely? This problem demands attention since the *SB* theory of identity is assumed to be an advance on the *Bgs* account of identity and so is supposed to provide something over and above what *Bgs* explains, with the addition of better explanations of new facts. But if they are different, then it follows that the new theory of identity of *SB* cannot cope with evolutionary informativeness; and I do not think that this is the case. Second, why should we hold that in *Bgs* Frege is concerned only with evolutionary informativeness, since the aim of *Bgs* is precisely to secure mathematical proof? Dickie does not provide any argument in this regard. Third, why should we consider Frege in *SB* to be concerned with rational informativeness alone, since from the start he formulates the whole discussion in epistemic terms regarding mathematical knowledge, aprioriticity and cognitive value (*Erkenntniswert*)? Here again, Dickie does not address this issue. So, although Dickie's analysis of informativeness seems very interesting and promising in the overall context of Frege's works, it is still difficult to understand why Frege has changed his views on identity.

Another series of papers devoted to this subject¹¹ approaches the problem from a different angle. Both Robert May and Richard Heck try to understand and

¹⁰ Dickie, "Informative Identities."

¹¹ Richard G. Heck, "The Julius Caesar Objection," *Language, Thought, and Logic: Essays in Honour of Michael Dummett*, ed. Richard G. Heck (Oxford: Oxford University Press, 1997),

Frege on Identity. The Transition from *Begriffsschrift* to *Über Sinn und Bedeutung* explain Frege's account of identity and, more generally, his semantics from the perspective of his work in the foundations of mathematics. I am sympathetic with their approach, yet I think that they err in what they think was Frege's mathematical setting responsible for this change. Basically, their main claim is that Frege's logicist thesis is in fact responsible for the introduction of the sense/reference distinction. As we'll see shortly, this is only partially true.

2. Identity

Logically speaking, we can distinguish two notions of identity. One is *numerical identity* or identity proper, and states that if "a is identical with b," then, in fact, 'a' and 'b' are just different names for the same object; a and b are the same under all aspects. However, we may use a second notion of identity: a and b are 'identical' only under one or some aspects, but not all. "Peter had an accident and his car was totally destroyed. But he went out and bought the same car" means that he bought the same model of car, but not the numerically identical one. The latter is sometimes called *qualitative identity*, whereas the former, by contrast is *quantitative identity*.¹²

This distinction is acknowledged in *Bgs* by the use of two different signs. '=¹', as in "3 x 7 = 21," means mathematical equality, whereas the second is '≡', and is defined in §8 as 'identity of content.' It is unlikely that the introduction of the latter is just a regrettable lack of rigor, since the main purpose of *Bgs* is to provide an *exact language* suitable for doing exact science. Thus, we have a formal sign for mathematical equality (=), which, from a logical point of view, is just a variant of qualitative identity, and a sign for 'identity of content' (≡), which is numerical identity. Yet, in the domain of numbers, the difference between them vanishes and later Frege will drop this notation and acknowledge that in mathematics

273-308; Robert May, "Frege on Identity Statements," in *Semantic Interfaces: Reference, Anaphora, and Aspect*, eds. Carlo Cecchetto, Gennaro Chierchia, and Maria Teresa Guasti (Stanford: CSLI Publications, 2001), 1-61; Richard G. Heck, "Julius Caesar and Basic Law V," *Dialectica* 59, 2 (2005): 161-178; Richard G. Heck, "Frege and Semantics," in *Essays on Frege's Conception of Truth*, ed. Dirk Greimann, *Grazer Philosophischen Studien* 75 (2007): 27-63; Richard G. Heck, Robert May, "Frege's Contribution to Philosophy of Language," in *The Oxford Handbook of Philosophy of Language*, eds. Ernest Lepore and Barry C. Smith (Oxford: Oxford University Press, 2006), 3-39.

¹² Note that in *Methods of Calculation based on an Extension of the Concept of Quantity* Frege calls 'quantitative identity,' what here is called 'qualitative identity.' However, as Frege himself will later acknowledge, in mathematics, in the pure quantitative domain, we should regard equality (identity under the quality of quantity) as numerical identity or identity proper. I shall discuss this in detail very shortly.

equality must be interpreted as numerical identity. Already at the time of *Grundlagen*¹³ (hereafter, *Gl*), he seemed to realize this, but it emerges explicitly in *Grundgesetze*¹⁴ (hereafter, *Gg*):

The primitive signs used in my *Begriffsschrift*, are to be found again here with one exception. Instead of the three parallel lines I have preferred the ordinary sign of equality [*Gleichheit*], since I have convinced myself that it has in arithmetic precisely the *Bedeutung* that I wish to designate [*bezeichnen*]. I use, that is, the word ‘equal’ [*gleich*] with the same *Bedeutung* as ‘coincident with’ [*zusammenfallend mit*] or ‘identical with’ [*identisch mit*], and this is also how the sign of equality is actually used in arithmetic. The objection that might be raised to this will probably rest on an inadequate distinction between sign [*Zeichnen*] and what is designated [*Bezeichnetem*]. Admittedly, in the equation ‘ $2^2 = 2 + 2$ ’ the left-hand sign is different from the right-hand sign; but both designate [*bezeichnen*] or refer to [*bedeuten*] the same number.

However, this issue raises the following question: why didn’t Frege use numerical identity from the very beginning? What prevented him from thinking of ‘ \equiv ’ as ‘ $=$ ’? One possible answer may concern the general purpose of *Bgs*: as a language suitable for general science, a *begriffsschrift*¹⁵ is not limited to mathematics and should somehow capture both notions of identity. For example, in physics or chemistry, one oxygen atom is identical with another, but this only means that they are qualitatively identical as atoms; as objects they are distinct and numerically different. However, this applies to spatio-temporal objects. Mathematical objects as numbers are for Frege logical objects, and are thus not constrained by any spatio-temporal limitation. Thus, in the domain of numbers equality may be seen as identity proper. The following passage from *On the Concept of Number* is very suggestive:

I cannot repeat the substance of my *Grundlagen* here. (...) There are various designations for any one number. It is the same number which is designated by “1+1” and “2”. Nothing can be asserted of 2 which cannot also be asserted of 1+1; where there appears to be an exception, the explanation is that the signs “2” and “1+1” are being discussed and not their content. It is inevitable that various signs

¹³ Gottlob Frege, *Die Grundlagen der Arithmetik, eine logisch-mathematische Untersuchung über den Begriff der Zahl* (Breslau: W. Koebner, 1884) translated as Gottlob Frege, *The Foundations of Arithmetic*, trans. J.L. Austin, 2nd ed. (Oxford: Blackwell, 1953).

¹⁴ Gottlob Frege, *Grundgesetze der Arithmetik*, 2 vols. (Hildesheim: Olms, 1962); preface, introduction and sections 1-52 of vol. I and appendix to vol. II translated in Gottlob Frege, *The Basic Laws of Arithmetic: Exposition of the System*, ed. Montgomery Furth (Los Angeles: University of California Press, 1964); this quote is from *The Frege Reader*, 197.

¹⁵ By the upper case italics ‘*Begriffsschrift*’ or *Bgs*, I refer to Frege’s well-known work, whereas the lowercase ‘*begriffsschrift*’ stands for the formal language presented in *Bgs* and *Gg*.

should be used for the same thing, since there are different possible ways of arriving at it, and then we first have to ascertain that it really is the same thing we have reached. $2 = 1+1$ does not mean that the contents of '2' and "1+1" agree in one respect, though they are otherwise different; for what is the special property in which they are supposed to be alike? Is it in respect of number? But two is a number through and through and nothing else but a number. This agreement with respect to number is therefore the same here as complete coincidence, identity. What a wilderness of numbers there would be if we were to regard 2, 1+1, $3 - 1$, etc., all as different numbers which agree only in one property. The chaos would be even greater if we were to recognize many noughts, ones, twos, and so on. Every whole number would have infinitely many factors, every equation infinitely many solutions, even if all these were equal to one another. In that event we should, of course, be compelled by the nature of the case to regard all these solutions that are equal to one another as one and the same solution. Thus the equals sign in arithmetic expresses complete coincidence, identity.¹⁶

However, in this context, another problem arises, and it may be seen as a second possible answer to the question of why equality and identity are different in *Bg*. This is the problem concerning the content of mathematics, which is a very serious problem for Frege. The formalists may reply that if mathematical equalities express logical identities, then all mathematics collapses into assertions such as $a = a$. But this would be unacceptable for Frege, given his firm conviction that mathematics has an objective content and is not a mere game with signs. The formalist could argument runs as follows: let us assume that mathematics is contentful, and that there is a difference between sign and thing signified, so that mathematics is not about signs but about the objects they signify. In this case, mathematical equalities state identities among numbers as the objects signified by mathematical signs. But then, if mathematical equalities are true numerical identities, all mathematics collapses to the logically uninformative principle of identity. So, in what sense is mathematics contentful, when everything reduces to the contentless " $a = a$ "? This is a serious objection, which must be addressed. Mathematical signs stand for mathematical objects and reference to such objects gives content to mathematics. But how can Frege hold these two apparently incompatible positions? Certainly, as Frege himself acknowledges, this is possible only with the help of his distinction between sense and reference, for one can now reply that mathematical equalities are true numerical identities which state relations within the realm of reference, yet hold that they are informative,

¹⁶ Gottlob Frege, *Posthumous Writings*, trans. Peter Long and Roger White (Oxford: Blackwell, 1979): 85-6.

because the terms of the identity have different senses, and this marks their difference in cognitive value.

The knowledge that the Evening Star is the same as the Morning Star is of far greater value than a mere application of the proposition “ $a = a$ ” – it is no mere result of a conceptual necessity. The explanation lies in the fact that the senses of signs or words (Evening Star, Morning Star) with the same *Bedeutung* can be different, and that it is precisely the sense of the proposition – besides its *Bedeutung*, its truth-value – that determines its cognitive value.^{17,18}

In sum, the *Bgs* view of identity has been modified not because of its alleged incapacity to deal satisfactorily with the problems generated by Millian views on proper names – as is commonly held in the literature surrounding this topic – but because of its role in the elaboration of Frege’s contentful mathematics thesis.¹⁹ In a footnote of *Gl* (§91) Frege says explicitly that the *begriffsschrift* is “designed, however, to be capable of expressing not only the logical form, like Boole’s notation, but also the content of a proposition.” The *content* – and this is consistent with its further splitting into sense and reference – is then seen as substantial information about the world, information that is ‘carried’ in the course of inference. So, a *begriffsschrift* has a dual role: to prevent the infiltration of subjective elements into the deductive chain of any scientific endeavor, and to carry information (about the world), information that is encapsulated in the structure of mathematical statements. Mathematical statements appear usually in the form of equations, equations constructed with the help of the equality sign ‘=’, hence the importance of identity. Mathematical statements involve signs which designate numbers, and hence *the importance of a clear account about the*

¹⁷ The passage continues with a three-point characterization which obviously applies to Frege’s account as well: “It follows from Dedekind’s quoted remark that for him numbers are not signs, but the *Bedeutungen* of signs. These three points:

- the sharp distinction between sign and its *Bedeutung*,
 - the definition of the equality sign as the identity sign,
 - the conception of numbers as the *Bedeutungen* of number signs, not as the signs themselves,
- hang most closely together and place Dedekind’s view in the starkest contrast to every formalist theory, which regards signs or figures as the real objects of arithmetic” (*Gg*, II, §138; in *The Frege Reader*, 271).

¹⁸ *Gg*, II, §138; in *The Frege Reader*, 271

¹⁹ This thesis is given by Frege’s strong claim that mathematics (contrary to *formalism* has content, and that this content (contrary to *psychologism*) is objective. ‘Formalism’ represents the position which claim that mathematics is nothing more than a mere game with empty signs, whereas ‘psychologism’ should be read as the claim that mathematical statements have and irreducible subjective content.

Frege on Identity. The Transition from *Begriffsschrift* to *Über Sinn und Bedeutung*
*mechanism of denotation in the case of proper names.*²⁰ Note that almost all I have
said about identity so far is nicely expressed by Frege himself in a letter to Peano:

[T]his does not yet explain how it is possible that identity should have a higher
cognitive value than a mere instance of the principle of identity. [...]

*At this point my distinction between sense and meaning comes in in an
illuminating way. [...]*

So nothing stands in the way of my using the equals sign as a sign of identity.²¹

It is clear that Frege's distinction between sense and reference plays an
illuminating role in showing, against (the counterattack of) the formalists, that in
mathematics we should take equalities as identities; yet mathematical statements
are *contentful* and not "boring instances of this boring principle [of identity]." Moreover, this passage also indicates the context in which we should understand Frege's concern about the puzzling nature of identity statements. Accordingly, it is important to have a unique and clear understanding of equality in mathematics as identity; for equality is a central concept, and if we hold that mathematics has content, then of course this content should be correctly displayed by mathematical equations.²² Thus, the moral of the story so far is that the motivation to show that identities like $a = b$ are informative and not just simple reiterations of the principle of identity is given by Frege's intention to establish in opposition with the formalists, that mathematical equations are capable of being substantive identities which enlarge our knowledge.

3. Identity and the sense-reference distinction

As we have seen, for Frege, mathematical equality is numerical identity, and two reasons seem to justify this step. The first is that it allows for a greater degree of unification and coherence in mathematics. The second concerns his struggle against the formalists: since mathematical statements are about objects, and since arithmetic includes equations, mathematical equality should therefore be taken to express a relation of identity between objects. But logical identity is just such a

²⁰ *Proper names* should be taken here in a broad sense which includes all singular terms.

²¹ Gottlob Frege, *Philosophical and Mathematical Correspondence*, trans. Hans Kaal (Oxford: Blackwell, 1980): 126-8; my italics.

²² *Gg*, vol II, §58, note A: "If mathematicians have divergent opinions about equality, this means nothing less than that mathematicians disagree as to the content of their science; and if we regard science as essentially consisting of thoughts, not of words and symbols, it means that there is no united science of mathematics at all – that mathematicians just do not understand one another. For almost all arithmetical propositions, and many geometrical ones, depend for their sense, directly or indirectly, upon the sense of the word 'equals'." In *The Frege Reader*, 261.

relation. Of course, now he has to counter a further attack of the formalists, namely that if mathematical equality is numerical identity, then most of mathematics collapses to mere instances of the cognitively uninteresting principle of identity. But for Frege, even though an equation states that we have an identity of *Bedeutungen*, their *Sinne* are different and so we can see why they are not ‘boring’ identities. All this is made explicit by Frege in the previously cited letter to Peano in which he affirms explicitly that his S/R distinction “comes in in an illuminating way” in explaining how “it is possible that identity should have a higher cognitive value than a mere instance of the principle of identity.”

It is clear now that the S/R distinction plays an important role in the ‘unified’ theory of identity, which is in fact more than just a mere unification of symbolization on new semantic grounds; rather it expresses the view that equations are about objects and are often informative identities. Thus, I find quite problematic the following characterization of Heck & May²³:

Though its application to identity-statements is extremely significant, it’s important to observe that the distinction between sense and reference does not emerge from any particular concern with identity-statements. At the time of *Begriffsschrift*, Frege treated mathematical equality as a notion distinct from ‘identity of content,’ the latter being the notion governed by Leibniz’s Law. But Frege must quickly have realized that the view is incompatible with a central tenet of logicism, namely, that there are no arithmetical notions with irreducibly mathematical content.

It is true that Frege was concerned with identity statements in mathematics. But his concern was to address the formalist criticism that if mathematical equalities are taken to be objectual identities, mathematical equations are no more informative than the principle of identity. But equality in *Bgs* is qualitative identity – not an ‘irreducible mathematical notion,’ but a special kind of logical identity. Having two kinds of identities at the time of *Bgs* wasn’t a threat to logicism since both could be seen as ‘logical’ and thus arithmetic is still logic. Frege adopted the ‘objectual identity view’ in order to cope with the requirement of his *contentful mathematics thesis*, namely that arithmetic has objective content and thus is about logical objects. Therefore, (arithmetical) identities should be informative and so they should be more substantive than the mere principle of identity. Thus, $a = b$ tells us more than $a = a$, and this ‘tells us more’ is nicely explained by Frege with the help of his S/R distinction.

[I]dentities are, of all forms of proposition, the most typical of arithmetic. It is no objection to this account that the word ‘four’ contains nothing about Jupiter or

²³ Heck, May, “Frege’s Contribution to Philosophy of Language,” 22.

Frege on Identity. The Transition from *Begriffsschrift* to *Über Sinn und Bedeutung*

moons. No more is there in the name 'Columbus' anything about discovery or about America, yet for all that it is the same man that we call Columbus and the discoverer of America. (*Gl*, §57)²⁴

It is interesting that "Columbus is Columbus" and "Columbus is the discoverer of America" express different thoughts, and thus have different cognitive values. Moreover, this means also that the senses of the two expressions, 'Columbus' and 'the discoverer of America,' are different, which certainly contrasts with the common interpretation of Frege as a strict descriptivist. Also, it may be objected that since this is a paragraph from *Gl*, and at that time Frege did not draw his S/R distinction, it may be somehow anachronistic to judge the issue in these terms. However, on the one hand it is clear that his views were basically the same, and the acknowledgement of his famous distinction was just a semantic refinement of the same view about the world. On the other hand, the following passage shows explicitly that, after the distinction has been introduced, he thought of things in exactly the same terms:

So the two signs are not equivalent from the point of view of the thought expressed, although they designate the very same number. Hence I say that the signs '5' and '2+3' do indeed designate the same thing, but do not express the same *sense*. In the same way 'Copernicus' and 'the author of heliocentric view of the planetary system' designate the same man, but have different senses; for the sentence "Copernicus is Copernicus" and "Copernicus is the author of heliocentric view of the planetary system" do not express the same thought.²⁵

So, mathematical equations and logical definitions are grounded on identities, and thus they express the fact that on either side of the equation we have different names for the same object. But do we have an identity of sense as well? That, certainly, would make all statements involving identity analytic statements, but in this way we cannot explain the fact that mathematics is contentful.

Summing up, the focus of this paper has been on a new interpretative perspective: Frege's original contributions, especially those on language and semantics, have been viewed from the perspective of his philosophy of mathematics. My claim is that the fact that *mathematics is contentful* is the true key to a better understanding of Frege's insights and results. From this perspective the connection between Frege's views on language and mathematics are seen as an organic whole, and so the role of Fregean senses in his overall project becomes clear. Frege's accounts of identity arose in the context of his struggle against formalism and psychologism, and thus it should be clear now that he introduced

²⁴ In Frege, *The Foundations of Arithmetic*, 69.

²⁵ In Frege, *Posthumous Writings*, 225.

Valentin Sorin Costreie

the S/R distinction in the framework of securing the contentful mathematics thesis. Thus, Fregean senses are not just the outcome of a mere linguistic analysis, rather they play an important role in the articulation of Frege's program in the foundations of arithmetic.