ON THE EPISTEMOLOGY OF PLATO'S DIVIDED LINE

Nicholas RESCHER

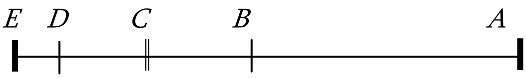
ABSTRACT: In general, scholars have viewed the mathematical detail of Plato's Divided Line discussion in Republic VI-VII as irrelevant to the substance of his epistemology. Against this stance this essay argues that this detail serves a serious and instructive purpose and makes manifest some central features of Plato's account of human knowledge.

KEYWORDS: Plato, Plato's epistemology, Divided line, Platonic Section, Proportionality

1. The Divided Line and its Divisions

In Book VI of his classic dialogue, *The Republic*, Plato contemplated four types of objects at issue in inquiry and cognition: ideals or ideas (such as perfect beauty, justice, or goodness);[1] mathematical idealizations (such as triangles, circles, or spheres); mundane, visible objects made by nature or man; and mere images, such as shadows and reflections. For abbreviative convenience we shall refer to these Platonic types as ideas, mathematicals, sensibles, and images, respectively.

With this classification in view, Plato proceeded to envision our knowledge about the world in terms of an arrangement which stands roughly as follows:



In setting this out he proceeded as follows:

> Suppose you take a line [EA], cut it into two unequal parts [at C] to represent, in proportion, the worlds of things seen [EC] and that of things thought [CA], and

[1] Contemporary discussants often call these *forms*. But a rose by any other name...

> then cut each part in the same proportion [at D and B]. Your two parts in the world of things seen [ED and DC] will differ in degree of clearness and dimness, and one part [ED] will contain mere [sensory] images such as, first of all shadows, then reflections in water then surfaces which are of a close texture, smooth and shiny, and everything of that kind, if you understand.[2]

The realm of ideas is generated and organized under the aegis of a supreme agency, the Idea of the Good. In the lead-up to the discussion of the Divided Line in book VI of the *Republic*, Plato (or, rather, his protagonist Socrates) acknowledges (506d-e) his incapacity to expound the Idea of the Good itself, instead mainly explaining its role in accounting for certain consequences, its "offspring" (*ekgonos*) and the "highest studies" (*mathêmata megista*, 504A) that provide a pathway towards it. And this path, so he maintains, can be illustrated by means of that diagramatic line. And Plato's Socrates then goes on to explain that in moving along a line from the mundane to the ideal we have the following situation:

> In the first part [*EC*] the soul in its search is compelled to use the images of the things being imitated [that lie in *DC*]... In the second part [*CA*], the soul passes from an assumption to a first principle free from assumption, without the help of images which the other part [*EC*] uses, and makes its path of enquiry amongst idealizations themselves by means of them alone. (510B)

Plato correspondingly distinguished between the visible "things of the eye" (things seen, *horata*) and the intelligible "things of the mind" (things thought, *noêta*). Preeminent in the later category are the "ideas" or "forms" (*ideai*) that provide the model or prototype (*paradeigma*) conformity to which constitutes things as the kind of thing they are. Yet not these ideas alone, but also the mathematical idealizations have a paramount role in the realm of intelligibles:

> When geometers use visible figures and discuss about them, they are not thinking of these that they can see but rather the ideas that these resemble; a square *in itself* is what they speak of, and a diameter *in itself*, not the one they are drawing . . . What they seek is to see those ideas which can be seen only by the mind. (510D)

Plato accordingly divided his line of cognition into two parts that represent the intelligible and the visible realms, and then divides each of these into two parts, higher and lower, each dealing with a correlative sort of object, as follows:

[2] Plato, *Republic*, 509D. Henceforth otherwise unspecified references are to this dialogue.

I. "Intelligibles"

1. Higher : ideas (*AB*)

2. Lower : mathematicals (*BC*)

II. "Visibles"

1. Higher : sensibles (*CD*)

2. Lower : images (*DE*)

Display 1
HOW CAPACITY CONCERNS DIFFER

Capacity	*Mode of Cognition*	*Objects*	*Temporal Aspect*	(*Mundane Spatio-Physical Aspect*)	
aisthesis	*eikasia* (supposition conjecture or imagination)	Images (*eikones*)	Fleeting	Present	Sensible Domain
aisthesi	*pistis* (observation-based conviction or belief)	Sensibles (*aisthêta*)	Transitory	Present	
logos	*logos dianoia* (rationcination or discursive thought)	Mathematicals (*mathêmatika*)	Unchanging	Representable	Intelligible Domain
nous	*epistêmê* (rational insight or reason)	Ideas (*ideai*)	Timeless	Absent*	

*NOTE: What is here called *mathematicals* may encompass symbolically mediated thought in general. While physical objects such as diagrams and counters ("calculi") can represent mathematicals, the physical world's objects only "participate" in ideals and cannot *represent* them. Participation reaches across a wider gap than does representation.

The corresponding ontology is thus dualistic, contemplating two realms, the changeable and the unchangeable. However, the epistemology is quadratic, contemplating higher and lower modes of knowledge with respect to either category.

As Plato saw it, what is pivotal with each of these four cognitive capacities in their relation to spatio-temporal issues can be indicated on the lines of Display 1.[3] The four modes of cognition at issue thus differ in standing and status. At the top of the scale stand the Ideas – the timeless ultimates of Platonic concern. As G.W. Leibniz was to put it:

> The Platonists were not far wrong in recognizing four kinds of cognition of the mind... conjecture, experience, demonstration, and [finally] pure intuition which looks into the connections of truth by a single act of the mind and belongs to God in all things but is given us in simple matters only.[4]

At the very bottom of the scale stand the "images" (*eikones*) at issue in suppositions based on the fleeting and superficial seemings of things: "shadows, reflections in pools and hard, smooth and polished surfaces, and everything of that sort" (510A).[5] The formal deliberations of ratiocination and the concrete observations that ground our convictions about the world's objects come inbetween.

As regards the mathematicals, there is an instructive passage in a critique of Plato in Aristotle's *Metaphysics*.

> Besides the Sensibles (aisthêta) and the Forms (ideai) he says that there are mathematicals (mathêmatika). These, so he says, are intermediate (metaxa) differing from the Sensibles in being eternal and immutable and from the Forms in that there are many like instances whereas the form itself is in each case unique. (Metaphysics 987b, 14-18).

Presumably one must construe this as saying, in effect, that an individual Idea/Form is a single unique unit, despite there being a plurality of concrete particulars that participate in it. But a geometrical shape, for example a circle, has

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[3] Plato uses the term *hexis*, i.e., capacity or skill or facility involved with a certain practice, what translators often render as *facility* (Greek *dianamis*) a terms which, in this context, awaits Aristotle. But for dianamis in the sense of power see 509B.

[4] Gottfried Wilhelm Leibniz, *Philosophical Papers and Letters*, ed. Leroy E. Loemker (Dordrecht & Boston: Springer, 1969), 593. [Letter to M. G. Hansch on the Platonic Philosophy: 25 July 1707.]

[5] For lucid accounts of *eikasia* see H. J. Paton, "Plato's Theory of *Eikasia*," *Proceedings of the Aristotelian Society*, 22 (1921/22): 69-104, and D. W. Hamlyn, "*Eikasia* in Plato's *Republic*," *The Philosophical Quarterly*, (1958): 14-23.

many abstract representations (differing in diameter, say), which are not concrete – though admitting of concrete participants in their turn.[6]

In summarizing the Divided Line discussion, the *Republic* has it that one should:

> Accept the four response-capacities (pathêmata) of the soul as corresponding to those four sectors: rational insight (noêsis) as the highest, ratiocination (dianoia) as the second, conviction (pistis) as the third, and supposition (eikasia) as the last; and arrange them proportionately, considering that they involve clarity (saphêneia) to the extent that the objects involve actual truth (alêtheia). (511E)

As Display 2 indicates, Plato's translators have used a wide variety of rendering for the four Platonic faculties. While I believe my own translations come closest to what Plato has in mind, I think that the time has passed for every discussant to introduce his own terminology. And so while I myself believe that the best nomenclature would be:

Rational Insight/Ratiocination/Conviction/Supposition

nevertheless, in the interests of impartiality, I think that we can live with the majority-rules reading of:

[6] On the *mathematika* see Jürgen Mittelstrass, "Die Dialektik und ihre wissenschaftlichen Vorübungen," in *Platon: Politeia*, ed. Otfried Höffe (Berlin: Akademie Verlag, 1997), 229-49. Admittedly, Myles Burnyeat is quite right, the *Republic* leaves the question of the ontological status of the mathematicals "tantalizingly open." [Myles F. Burnyeat, "Plato on Why Mathematics is Good for the Soul," in *Mathematics and Necessity: Essays in the History of Philosophy,* ed. Timothy Smiley, *Proceedings of the British Academy*, 103 (2000): 22]. However, see also Christopher Gill, "Plato, Ethics and Mathematic," in *Plato Ethicus: Philosophy is Life: Proceedings of the International Colloquium Piacenza*, eds. Maurizio Migliori, Linda M. Napolitano Valditara and Davide DelForno (St. Augustin: Academia Verlag, 2004),165-75; and Christopher Gill, "The Good and Mathematics," in *Pursuing the Good: Ethics and Metaphysics in Plato's Republic*, eds. Douglas L. Cairns, Herrmann Fritz-Gregor, and Terry Penner (Edinburgh: Edinburgh University Press, 2007).

Intellect//Thought//Belief//Imagination

Display 2
DESIGNATION FOR THE PLATONIC CAPACITIES

epitêmê//*logos*//*pistis*//*eikasia*
rational insight//ratiocination//conviction//supposition
intuition//demonstration//belief//conjecture[7]
intelligence//thinking//belief//imagining[8]
reason//understanding//belief//imagination[9]
reason//understanding//belief//conjecture[10]
intelligence//understanding//faith//conjecture[11]
intelligence//thinking//belief//illusion[12]
intelligence//thought//conviction//conjecture[13]
understanding//thought//confidence//imagination[14]
understanding//thought//belief//imagination[15]
intellect//thought//trust//fancy[16]

On this basis, every polled interpreter gets to have *something* their own way excepting – alas! – myself. Still, for the present I shall sink my own preferences in deference to the common good.

Be the issue of terminology as it may, the fact remains that a definite four-rung ladder is at issue here, which conjointly characterizes both a type of knowing and a grade of knowledge. In ascending order these four are: superficial

[7] William Whewell, *The Philosophy of Discovery* (London: Parker & son, 1860).
[8] F. M. Cornford, *Plato's Theory of Knowledge* (London: Kegan Paul, 1935).
[9] Anders Wedberg, *Plato's Philosophy of Mathematics* (Stockholm: Almqvist & Wiksell, 1955).
[10] W. H. D. Rouse, *Great Dialogues of Plato,* trans. W. H. D. Rouse, eds. E. H. Warnaugh and P. G. Rouse (New York and Scarbourough, Ontario: New American Library, 1956).
[11] John Malcolm, "The Line and the Cave," *Phronesis*, 7 (1962), 38-45.
[12] R. C. Cross and A. D. Woozley, *Plato's Republic: A Philosophical Commentary* (Basingstoke: Macmilllan, 1964).
[13] Richard Robinson, *Plato's Earlier Dialectic* (Oxford: Clarendon Press, 2nd ed. 1984).
[14] Gail Fine, "Knowledge and Belief in Republic V-VII," in *Epistemology*, ed. Stephen Everson (Cambridge: Cambridge University Press, 1990).
[15] G. M. A. Grube, ed., *Plato's Republic* (Indianapolis: Hackett, 1974).
[16] Nicholas Denyer, "Sun and Line: The 'Role of the Good'," in *The Cambridge Companion To Plato's Republic*, ed. G. R. F. Ferrari (Cambridge: Cambridge University Press, 2007), 284-309.

inspection (*eikasia*), observation (*pistis*), mathematically informed understanding (*dianoia*), and rational insight (*epistêmê*). Here mind-managed *dianoia*, formal reasoning based on mathematics and logic, is seen as a more powerful cognitive intrumentality than anything that the senses have to offer us. But at the very top of the scale stands *epistêmê*, the authentic rational knowledge characterized by Plato as unerring (*anmarêton*: 477A), access to which is possible through dialectical reasoning alone. And what renders *dianoia*/mathematics inferior to *noêsis*/ideatics is that mathematical reasoning still relies on images (diagrams) and hypotheses while the methods of dialectic involve no such "contaminating" compromises with an inferior resource.

Those four Platonic capacities are not different stages of learning, let alone "stages of mental development." Rather they represent different sorts of knowledge that offer increasingly more accurate insight into the nature of True Reality.[17] Stocks[18] maintained that Plato subscribed "an old assumption, prevailed among the Greeks, [namely] that differences of apprehension must be due to differences of the apprehended."[19] There is, however, no reason to saddle Plato with the idea that different capacities *must* deal with different sorts of object, but only that they *can* do so. In specific, those "higher" capacities need not deal with a higher class of objects: it is just that they can do so on occasion.

As Plato thus sees it, fundamentally different sorts of cognitive processes are at work and they can relate to different sorts of things as their products. Overall, the matter stands as per Display 3. Accordingly, the question "Does the Divided Line discussion deal with process (modes of cognition) or with product (objects of cognition): does it deal with ontology or with epistemology?" has to be answered by saying: both! But at least in the first instance the issue is one of different modes of knowing rather than different topics of knowledge. All are addressed to one selfsame object, Reality, but they deal with it with very different degrees of clarity and adequacy.

Now the Divided Line narrative has it that a certain proportionality obtains uniformly throughout these divisions, as represented by the dual proportions:

$$\mathrm{I} : \mathrm{II} :: \mathrm{I}_1 : \mathrm{I}_2 :: \mathrm{II}_1 : \mathrm{II}_2$$

[17] However, on this dogmatic view of the matter see H. W. D. Joseph, *Knowledge and the Good in Plato's Republic* (London: University Press, 1948), who covers a wide range of opinion on the topic.

[18] J. L. Stocks, "The Divided Line of Plato Republic VI," *Classical Quarterly*, 5 (1911): 73-88.

[19] Stocks, "Divided Line," 76.

Thus overall, all of the following ratios (proportions) are all to be identical.

- opinion : knowledge (EC : CA)
- mathematical idealizations : ideal realities (CB : BA)
- appearances : perceptions (ED : DC)

Basic throughout is the crucial contrast between deep understanding (*gnôsis*) and mere superficial belief (*doxa*).

Display 3
PLATO'S VIEW OF COGNITIVE PROCESSES AND THEIR OBJECTS

Cognitive Resource or Capacity	*Process of Cognition*	*Products or Objects of Cognition*
I. KNOWING (*nous* or *gnôsis*) (*noêta*)	I. INSIGHT (*noêsis*)	I. INTELLIGIBLE THOUGHTS
1. Rational insight (*epistêmê*)	1. Intuitive grasps (*epistasis*)	1. Ideals and ideas, "Forms" (*ideai*, *gnôsta*)
2. Ratiocination (*dianoia*)	2. Formal reasoning (*dianoêsis*)	2. Mathematical Conceptions (*mathêmata*)
II. OPINING (*doxa)* [SENSING]	II. SENSORY APPREHENSIONS or (*aisthêta*)	II. SENSE JUDGMENTS (*doxasta*)
1. Conviction (*pistis*)	1. Observation (*horasis*) and more generally perception (*aesthesis*)	1.Observed Features (*horata*)
2. Conjecture and seeming (*eikasia*)	2. Imaging (*hêmoiôsis*)	2. Casual Appearances or "Images" (*phantasmata* or *eikona*)

The resultant situation is encapsulated in the line elaboration of Display 4.

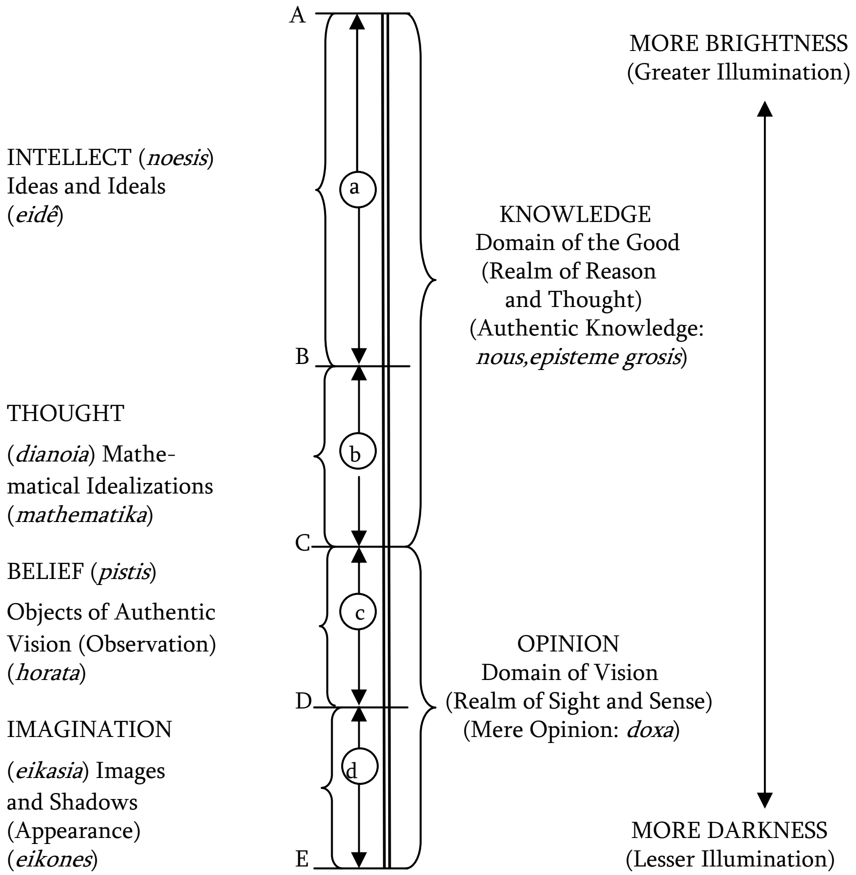
Against this background, the present discussion will implement a certain definite perspective and procedure. It proposes to take the Divided Line narrative seriously as it stands literally and not more than minimally figurative or metaphorical. And it then asks where this leads in regard to the larger issues of Plato's epistemology. So where most discussants have asked what Plato's epistemology means for the Divided Line, the present discussion proposes to reverse this interpretative line.

2. What do Those Proportions Represent?

The starting point of the line of thought at work in Plato's account is the idea of a relational comparison or analogy based on the pattern:

- Even as X is to Y in point of ϕ so also Z is to W in point of ϕ.

Display 4
PLATO'S DIVIDED LINE



On this basis, for example, the "ship of state" analogy would emerge roughly as follows:

- Even as a ship's people (crew and passengers) live under the aegis of a directive power (the captain) that is ultimately responsible for their well-being, so also do the people of a country live under the aegis of a directive power (the government) that is ultimately responsible for their well-being.

What is at issue in all such cases is an analogizing proportionality of the fact:

$X : Y :: Z : W$ in point of ϕ

Now whenever ϕ happens to be a feature that is *quantifiable*, then we are in a position to transmute the analogizing proportionality at issue into an outright mathematical equation:

$$\frac{X}{Y} = \frac{Z}{W}$$

The ruling idea of Plato's Divided Line is to exploit just this prospect of transmuting descriptive analogies into mathematical proportions Plato's Divided Line narrative transmutes what is a mere *analogy* (in our present sense) into a qualitative equation, an *analogon* in Aristotle's technical sense of "an equating (*isotês*) of ratios or proportions (*logoi*)."[20]

In its analytical role, the Idea of the Good mirrors the dual function of the sum in both providing the warmth that sensations organic life and the light by which existing things can be cognitively apprehended. On the cognitive side we reach the basic proportionality on which this process rests is:

Light: Objects of sight: :the Good: Ideas in point of ϕ

But what is ϕ to be in the Divided Line context? Clearly, it must be something that is quantifiable in order to provide for what can function as an outright proportionality-equation as per:

$$\frac{(Sun)\text{ - }Light}{Sight\text{ - }objects} = \frac{The\ Good}{Ideas}$$

And Plato has it that this is to be illustrative – preeminently daylight, the light of the sun. What is at issue here with illumination is increasing clarity of

[20] Aristotle, *Niomachean Ethics*, 1131a31. On Plato's handling of analogies see A. S., Ferguson, "Plato's Simile of Light Again," *Classical Quarterly*, 28 (1934): 190-210.

vision be it by ocular sight or mental insight – the sort of thing inclined with the locution "Ah, it is now clear to me!"

This matter of providing for a quantifiable respect ϕ is crucial to Plato's reasoning. And here he sees the key factor as one of accessibility of thought. It is, in sum, a matter of providing for insight, for intelligibility, namely the power of *illumination*. As the discussion of the role of the sun at 507A-509B makes clear, the role of sunlight in apprehension is to mediate between the mind and its object. Just as the sun provides the power of visibility (*ta to horasai dunamis*) [509B], so the Good provides the power of intellection (*ta to noêsei dunamis*). Those proportions at issue are thus to reflect the comparative extent to which we are given significantly informative insights from the resources afforded by the mode of cognition at issue. The basic idea is that just as – and to the same extent that – sunlight makes sight-objects accessible to the mind through vision (*horasis*) so the Good makes ideas accessible to the mind through reason (*noêsis*)?

Light, of course, contrasts with darkness. At 478C-D we are introduced to yet another factor: ignorance (*agnioia*), and told that "opinion (*doxa*) is darker than knowledge (*gnosis*) and brighter than ignorance." So ignorance (utter darkness), is at the bottom of the scale – "off the chart" so to speak. (And perhaps the Good is to be located similarly at the other end.)

The divided line with its pinnacle of knowledge regarding the Ideas is joined to the simile of the Sun, that offspring (*ekgonos*) and resembler of the Idea of the Good (506E). And what both have in common is of course the illumination that constitutes a requisite for seeing things, be it with the eye of the body or the mind's eye. Plato apparently holds that even as sunlight both reveals actual things and produces their shadows, so the intellect both reveals the Ideas and engenders the mathematical abstractions that are their mere reflections.

3. The Analogy of Light

The Divided Line narrative presents us with a trio of proportionalities since we are told that:

$$\frac{a}{b} = \frac{c}{d} = \frac{a+b}{c+d}$$

This is the formal substance of what might be called the Platonic Section. But just what do these proportions mean? What is it that those comparative line-lengths are supposed to represent? Regrettably, Plato does not really offer as much information about this as one might wish for. Pretty much all the guidance he provides is that the proportions are to reflect a differentiation in respect to reality and truth (*dihêrêsthai alêtheia te kai mê* [510A]).

Both the general context of the discussion, and the Cave Allegory in particular, make it clear that those Divided Line segments are intended to correlate with the cognate power to give insight, to make intelligible, to *illuminate*. The crux of the matter is how much the information of a certain sort contributes to a proper understanding of the nature of reality and our place in it. Length is to reflect the comparative cognitive significance or importance in the wider setting of our knowledge of reality.

The overall situation that the Divided Line account puts before us accordingly stands as per Display 4 above. At the top, the dazzling brightness of the Idea of the Good yields greater – but not *infinitely* greater – information than that of our mundane observation. And this gearing to illumination means that the different parts of the line will deal – at least in the first instance – not so much with different *kinds* of knowledge as with different *grades* of knowledge.

The Divided Line is seen to provide a conjoint illustration of a cluster of proportions that implement the analogy of light. For explaining the proportions at issue, Plato tells us that the length of each segment measures the comparative "clarity and obscurity" (*saphêneia kai asapheia*) or "intelligibility" (*alêtheia*)[21] of what is at issue – i.e., its comparative contribution to knowledge and understanding. To be sure there are many cognitive virtues: probability, informativeness, reliability, accuracy, detail, clarity. But none of these quite fill the bill. Instead, what seems paramount here is inherent in the simile of light: lucidity, illumination, insight, enlightenment. The model is the capacity for being seen that sunlight provides (*ta tou horasthai dunamis* [509B]). Something like profundity of understanding seems to be the issue – *illumination* or *enlightenment* (*phanos*) in short. Just this, we may suppose, is what Plato had in mind in speaking of "clarity and obscurity." And just as the sum is the cause (*aitia*) of visual observation so the Idea of the Good affords "the very brightest illumination of being," (*tountos to phanotaton* [518D]) in the realm of thought. This circumstance – that the line orders those faculties in point of cognitive power, and that the size of its segments reflects the amount of illumination achieved in the correlative domain has been pretty much agreed upon since antiquity.[22]

Gail Fine concluded her instructive study of Plato's epistemology by insisting that "Plato does indeed explicate *epistêmê* in terms of explanation and

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[21] See 509D and cf. 478C-D.

[22] Proclus' *Commentaries on the* Republic *of Plato* observes – as is Plutarch's view (in his *Platonic Questions*) – that in order of volume/quantity of information (rather than quality) one would have to reverse the size of the segments. See also Section 5 below and Denyer, "Sun and Line," 293.

interconnectedness, and not in terms of certainty or vision."[23] But this view of the matter is predicated on maintaining a sharp contrast between an holistic/coherentistic approach to knowledge and one that is based on insight and intellectual apprehension – between a discursive and an intuitive approach to cognition. And in taking this position one elides the prospect that the apprehension of explanatory interconnections is the fuel that energizes the interactive apprehension of certainties, so that the grasp of explanatory connections is the illuminative basis of intuitive certainty. One fails, in sum, to appreciate that discursive reasoning may open the door to intuitive insight. But it seems to be along just these lines that Plato saw the connection between illumination and inquiry. The cognitive level of authentic Knowledge (*epistêmê*) at issue in AB will always involve not just a certain fact but an explanatory rationale in which this certainty is grounded.[24] This sets the gold-plated standard by which the rest of our cognition must be judged. And illumination is the crux here since the mission of knowledge is to illuminate our way through this world's darkness to the conception of a good life as encapsulated in the Idea of the Good.

To be sure, it is not the formal structure of the line itself but the substance of the overall explanatory discussion that is going to be crucial.

It is clear that the proportions of the Platonic Section do not and by of themselves accomplish the job that the Divided Line Narrative is supposed to achieve. For all of the specified proportionality conditions are satisfied when $a = b = c = d = 1$. This circumstance line shows the justice of W. D. Ross' observation that "the line, being but a symbol, is inadequate to the whole truth which Plato meant to symbolize."[25] For clearly the idea that equal illumination is provided by Vision and by Reason is a non-starter for Plato who rejects prospect out from the outset (at 509D).

Just what is to be made of Plato's idea of illumination? It is clear, from what we are told, that even the image-mongering of mere "conjecture" (*eikasia*) provides *some* illumination and has some positive contribution to make. Granted, the illumination of the Good-illuminated Truth is vastly greater than that of the shadow-realm of mere images, but even this latter domain yields *some* illumination – and that of a magnitude that is proportionally limited to the magnitude of that former domain. However, in the quantitative perspective opened up by the Divided Line narrative, this contribution is comparatively very small.

Plutarch somewhat perversely suggested that the Divided Line narrative puts matters into reverse. As he sees it, shorter line segments would better reflect

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[23] Fine, "Knowledge," 115.

[24] *Meno* 98a, *Phaedo* 76b, *Republic* 531c, 534b.

[25] David Ross, *Plato's Theory of Ideas* (Oxford: Clarendon Press, 1951), 46.

coherence and unity of thought, while segments of greater length would better represent observability, indefiniteness, and of more obscure and less perspicuous knowledge.[26] But that's just not how Plato's account does it: his segments measure light rather than darkness. And even on the face of it, Plutarch's complaint that the Line should measure obscurity rather than illumination seems problematic. After all, with measurement of all sorts one accentuates the positive: one measures the weight of objects not their lightness, the duration of time and not its brevity, the height of persons not their shortness.

Overall, in coming to terms with the Divided Line narrative one must accordingly recognize:

1. What is at issue are not *items* of knowledge, nor yet bodies or *branches* of knowledge, but *types* of knowledge as defined by the *method of acquisition* at issue: respectively superficial inspection, sensory observation, ratiocination/ calculation, and dialectically developed insight. The focus is this less the product known than the process – the method of cognition that is at work.
2. What is at issue is not the *substance or theme* of the sort of knowledge in question, but its *significance or value*.
3. What is crucial in this valuation is neither the utility or applicative efficacy of the sort of true knowledge in question, nor yet the extent of time and effort needed for the mastery, but its illuminative strength: the extent to which it throw light on the condition of man in reality's scheme of things.
4. The highest form of knowledge is not thought, ratiocination and calculation, but rather the wisdom achieved in philosophy by the method of rational dialectic. However, even the world of shadows affords *some* instruction and enlightenment. While this is doubtless precious little, it cannot be set at nothing, even in a comparison to authentic *epistêmê*.

The key to the issue of Plato's perspective on cognition is that it is dialectic, the methodology of philosophizing, which stands at the forefront, and that philosophy – the queen of the sciences as it were – stands at the pinnacle. However, the proper assessment of the types of knowledge is a matter of proportion and harmony – the line and its proportionalities are plainly geometric and quantitative in nature. It would appear that in insisting on the philosophical importance of a mathematical informed view of things, Plato was putting his money where his mouth is in setting out the Divided Line narrative.

[26] Plutarch, *Platonic Questions*, 1001 d-e.

4. A Point of Contention: Did Plato Mean It? (Metaphor or Model?)

Scholars have long debated whether the Divided Line narrative is a mere flourish of literary ornamentation making the broad point that the realm of thought is superior to and more significant than the realm of sense, or whether something more substantial and significant is going on. Specifically, do those mathematical details matter? Some commentators have little patience with this entire Platonic exercise in mathematical epistemology. One recent discussant, for example, dismisses the fourfold division and its proportionalities with the breezy comment that they are "at best a framework on which to hang the comparison of mathematics and dialectic, [and] at worst an empty play with the idea of mathematical proportion."[27] (If this is critical elucidation, then what price is to be payed for obfuscation?)

To begin with, it should be acknowledged that W. D. Ross is right in saying that "the equality of *DC* to *CE*, though it follows from the ratios prescribed, is never [explicitly] mentioned."[28] Some see it as "an undesirable though unavoidable consequence of the condition, which Plato would have avoided if he had been able, and to which we should attach no significance"[29] with another commentator dismissing it as "as embarrassing detail."[30] But it would surely be unwise – as well as unkind – to fail to credit a geometer as sophisticated as Plato with recognizing consequences of his claims that would be at the disposal of a clever schoolboy. Anders Wedberg characterizes the equality of DC with CE as "obviously an unintended feature of the mathematical symbolism to which no particular significance should be attached."[31] One wonders who conducted the séance at which Plato informed Wedberg of its unintendedness?

The present discussion is predicated on the idea – the working hypothesis, if you will – the mathematical detail of Plato's discussion is to be taken seriously. It will thus be supposed that we are dealing not with some merely metaphorical analogy, but with a full-fledgedly mathematical description of man's cognitive situation. And we shall suppose that Plato, good geometer that he was, formed his account with intention aforethought – that it was not some random whim that

[27] Richard Robinson, "Hypothesis in the Republic," in *Plato: A Collection of Critical Essays*, ed. Gregory Vlastos (Notre Dame: University of Notre Dame Press, 1978), 193.

[28] Ross, *Plato's Theory,* 45.

[29] Paul Pritchard, *Plato's Philosophy of Mathematics* (St. Augustin: Academia Verlag, 1995), 91. Here Prichard does not speak *in propia persona*.

[30] John Gould, *The Development of Plato's Ethics* (Cambridge: Cambridge University Press, 1955), 31.

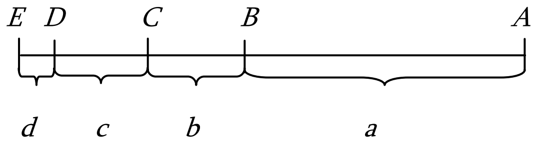
[31] Wedberg, *Plato's Philosophy*, 102-03.

philosophers should be geometricians (*mêdeis ageômetrêtos eisitô*),[32] and that Book VII of the *Republic* required that the training period in geometry for guardians be longer (indeed twice as long) as that in dialectical theory. Accordingly, the present deliberations will take the line that it is one of the salient tasks of an adequate interpretation to give some plausible account of why – and how – those quantitative relations would obtain on the basis of Platonic principles.

Approached from this angle, the pivotal problem becomes that of explaining just how it is that the various mathematical specifications that Plato incorporated into his Divided Line narrative function to inform his theory of knowledge and to account for its formative features. The interpreter of *Republic* VI-VII who leaves those proportions out of consideration is offering us Hamlet without the Ghost.

5. The Platonic Section

Proceeding in this direction, let us envision the idea of a Platonic Section based on a diagrammatic set-up of the format:



where, as already noted, the magnitudes at issue are subject to the following specified proportionalities:[33]

$$\frac{a}{b}=\frac{c}{d}=\frac{a+b}{c+d}$$

The unusual feature of the Platonic Section lies in its interrelating four quantities. This feature distinguishes it from the tripartite proportionality relations commonly treated in the Greek theory of proportions where, we find deliberations on such relations as

$\alpha : \beta :: \beta : \gamma$

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[32] However uncharismatic the letter of this observation, its spirit seems thoroughly Platonic.

[33] Rosemary Desjardines, *The Rational Enterprise: Logos in Plato's* Theaetetus (Albany, NY: State of New York University Press, 1990), 481, has it that not only $\frac{a}{b}=\frac{c}{d}$, but also that $\frac{a}{c+d}=\frac{b}{c}$ This second proposition looks to be without visible means of support. Compare Pritchard, *Plato's Philosophy*, 97.

characterizing the geometric mean: $\alpha\gamma = \beta^2$. However, just this relationship plays a pivotal role in Plato's Allegory of the Cave,[34] and we shall shortly address the implications of this difference.

The design of the Platonic Section has an array of significant mathematical consequences. These include:

$b = c$

$$\frac{a}{b} = \frac{c}{d} = \frac{b}{d} = \frac{c}{d}$$

The appendices provided below will examine this situation more closely.

Plato himself was fully explicit with regard to at least some of these consequences. Thus he tells us that in its efforts at understanding the Ideas (i.e., in *AB*) "The mind treats as mere images (i.e., as a *DE* analogon) those actual things (*CD*) which themselves have mere images in the visual realm (i.e., in *DE*)."[35] We are thus presented with the proportion:

$$\frac{AB}{CD} = \frac{CD}{DE} or \frac{a}{c} = \frac{c}{d}$$

The position is that to just the extent that those mere images (*DE*) convey some contention of the objective features of things (*CD*), so the mathematicals (*BC*) convey some indication of the order of ideality (*AB*).

6. Why Should It Be That *B* = *C*?

Republic 510A says that the Divided Line's segments represent "a division in respect of reality and truth" and not "in respect of *decreasing* reality and truth." Yet nothing *about the proportionalities* at issue conflicts with the prospect that various segments have equal length. (However, Plato does block this prospect by a specific stipulation *ad hoc* at 509D). However, it follows from the proportionalities of the Line that $b = c$. (See Appendix 1.) Moreover, Plato's text nowhere explicitly acknowledges that $BC = CD$ (i.e., $b = c$), and some commentators therefore think that "it may be indeed that he himself failed to notice that it was a consequence".[36]

[34] See Appendix 3.

[35] 511B.

[36] Cross and Woozley, *Plato's Republic,* 209. Raven speaks of it as "an unfortunate and irrelevant accident." (J. E., Raven, *Plato's Thought in the Making: A Study of the Development of his*

But could he really have been oblivious to this? Surely not! As Nicholas Denyer has rightly insisted, "Plato was too good a geometer for that."[37]

For Plato mathematically informed reasoning (*dianoia*) constitutes a mode of cognition superior to sense-based observation (*aisthesis/pistis*), seeing that it appertains to the intelligible rather than visible realm. On this ground, interpreters have been perplexed by the equating of *BC* and *CD* – the respective illumination afforded by those two cognitive resources. And such commentators have accordingly found it puzzling that a lower faculty should have as much to offer by way of cognitive illumination as a higher one. H. W. D. Joseph observes that "the second and third segments as equal: whereas if Plato had wished to set forth to prosper in four stages, he should have given us a continuous proportion in four terms."[38] And Denyer wonders how this "surprising equality" can be reconciled with Plato's view that cogitation/*dianoia* (Denyer calls it *thought*) outranks sense-belief/*pistis* (Denyer calls it *trust*).[39] So why should Plato hold that sensory *inspection* and mathematical *reflection* to be co-equal in point of illumination? However, such puzzlement fails to distinguish between process and product: a more powerful process need not necessarily yield a greater result; it could well provide a product of co-equal value more elegantly or effectively. After all, an electronic typewriter is a more powerful instrument than a pen, but whatever it can write can be written by a pen as well. An automobile is a morepowerful means of transport than Shank's mare, but wherever the former can take you, the latter also can (if you have the energy and time).

Through running together the Line and Cave, David Gallop depicts the line's parts as involving the distinction between waking and dreaming[40]:

A. the *noêsis* of dialectic: ("waking")
B. the *dianoia* of mathematic: ("dreaming")
C. the *horata* of the natural science: ("waking")
D. the *ekasis* of the plain man's observations: ("dreaming")

Metaphysics (Cambridge: Cambridge University Press, 1965), 145). And Pritchard notes that various commentators view this as "an undesirable though unavoidable consequence of the [specific] conditions, which Plato would have answered if he had been able, and to which we are to attach no significance" (Pritchard, *Plato's Philosophy*, 91).

[37] Denyer, "Sun and Line," 213.

[38] Joseph, *Knowledge and the Good*, 32.

[39] Denyer, "Sun and Line," 295.

[40] See David Gallop, "Image and Reality in *Plato's Republic*," *Archiv für Geschichte der Philosophie*, 47 (1965): 113-31.

But this raises real problems. In specific why should mathematical cognition be a mode of "dreaming"? And why should the "dreaming" of B yield every bit as much illumination as the "waking" of C? Most likely, rather different considerations are at work in the Line and the Cave accounts – as will emerge more clearly below.

And yet, the circumstance that while $b = c$, Plato himself does not make anything of this exerts a strange fascination on his interpreters. It emboldens them to think that they know Plato's thoughts better than the master himself. Thus J. E. Raven writes:

> As Plato's failure to mention the fact suggests, it is an unfortunate and irrelevant accident [that $b = c$]. Although it is a geometrical impossibility at once to preserve the [specified] proportions, which are all important, and to make each segment longer than the one below it, *this is what Plato had it been possible, would have wishes to so*[41] (my italics)

So quoth the Raven. But how can he possibly know?

Julia Annas endeavors to solve the problem by declaring that "Plato is not interested in having each section of the Line illustrate an increase in clarity ; his interest lies in internal studies of each [segment], not in the whole line that results."[42] Yet one cannot but wonder why, if the Line structure is indeed immaterial, Plato should go to considerable lengths to set it out. No sign of disinterest, that!

Morrison[43] maintains that "the contents of the two middle subsectors (i.e., *BC* and *CD*) are identical in the lower subsection (*CD*) they are used as originals and in the upper subsection (*BC*) used as likenesses." But this looks decidedly far-fetched. The crude diagram of the geometry-teacher at issue with visualization is surely not a *likeness* of the theoretical mathematician's abstract figure, but a crude representation (*eikon*) of it. The concerns of *dionoia* are a step upward from more vision towards the Ideas, not a retrogression from them towards the phantasms of *eikasia*.

Perhaps $BC = CD$ might obtain because everything in the world has a dual aspect: both a mathematically characterizable shape and a sense-provided qualitative texture. This idea is favored by Paul Pritchard who writes: "This much is clear, the objects in *DC* are the same as those in *BC* but now they are used as images of something else."[44] We are, that is, dealing with the same items regarded from different systemic points of view. And this perspective might well be grounded in

[41] Raven, *Plato's Thought*, 145.

[42] Julia Annas, *An Introduction to Plato's Republic* (Oxford: Oxford University Press, 1981), 248.

[43] J.S. Morrison, "Two Unresolved Difficulties in the Line and Cave," *Phronesis*, 2 (1977): 212-31.

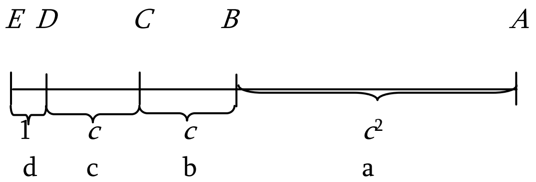
[44] Pritchard, *Plato's Philosophy*, 92.

Plato's Pythagorean inclinations. After all, Plato seems drawn to the Pythagorean precedent of holding that such cognitive grasp as we securely have upon the mundane actualities of the world is mediated by mathematics.[45] Accordingly, the guiding thought would be that the worlds realities of this world can be regarded either from the standpoint of empirical observation or from that of geometric analysis and that these approaches are of co-equal significance because each is informatively impotent without the other. Thus, equating *BC* and *CD* might be the result of the view that observation can only yield adequate illumination insofar as it can be mathematically rationalized. In other words: observation yields reliable information ("insight") only – but to exactly the same extent – that it is mathematically formalizable. In the end, then, it may be that the relationship should be as one of coordination and that here something of a Kantian perspective is called for: observation without theory is blind and theory without observation is empty. The data of sensory perception (*aesthesis*), are only illuminating where rigorous reasoning (*dianoia*) can make sense of them, and conversely *dianoia* cannot do its illuminative work without having the materials of *aesthesis* to address.

7. Why should it be that $a = b^2 = c^2$ (When $d = 1$)?

Analysis of the proportions at issue with the Divided Line, indicates that d, c, b¸ a stand to one another as per d, kd, kd, k^2d. We shall designate these correlations as the Whewell Relations because this situation was first noted and discussed by William Whewell in his 1860 *Philosophy of Discovery*.[46]

These relations have it that when we do our measuring in terms of d as a unit (so that $d = 1$), we are lead straightway to the result that $a = b^2 = c^2$. And this opens up some larger vistas. For it means that when we use d as our unit of measure, then the overall proportionalities of the Divided Line will stand as follows:



[45] On Plato's Pythagoreanism see Erich Frank, *Plato und die sogenannten Pythagoraer* (Halle: Max Niemayer, 1923, 2nd ed., Darmstadt: Wissenschaftlishe Buchgesellschaft, 1962).

[46] Whewell, *Discovery*, 444. See Appendix 2 below. There should be little wonder that Whewell had a firmer grip than most on the mathematics of the Divided Line, for alone among Platonic scholars he was a senior wrangler at Cambridge.

During 1860-80, the Whewell Relations were considered by several commentators,[47] but misunderstood by them as having d, c, b, a be 1, c, c^2, c^3 rather than 1, c, c, c^2 as the just-given diagram indicates. (This error was noted by Henry Jackson in his 1882 paper.[48]) Still, after the brief debate among the 19th century interpreters, the Whewell Relations simply dropped from sight. As far as I can see, no 20th century commentator has touched on these relations, and the question of their rationale remains in limbo.

They do, however, have interesting ramifications. Specifically, they mean when we measure length in d units (with $d = 1$), we have it that the overall length of AE is $1 + c + c + c^2 = 1 + 2c + c^2 = (1 + c)^2$. On this basis we can say that the total illumination available in AE is exactly the square of the mundane illumination provided by the senses (in CE). Accordingly, c alone – the measurement of illumination afforded by the senses – can be seen as the determinative factor for the illumination provided by cognition at large.

Can anything be said about the relationship of magnitude as between a and d? Not really! For the ratio $a : d$ is left wide open. Nothing about those Line proportionalities bears upon the size of the other segments in relation to DE. That c^2 is vastly greater than d is an *ab extra* supplementation to the postulated proportionalities, for – as already noted – nothing in the Platonic proportionalities prevents the prospect that $a = b = c = d$. (Clearly, these proportions tell only a part of the story!) After all, the Divided Line narrative must be construed in such a way that c is larger than d, and that in virtue of this a, which aligns with c^2, becomes really enormous. The illumination of mind sight is vastly greater than that of eyesight. In context – but only then – are those proportionalities are effectively designed to carry a significant lesson. And so, when Sidgwick cavalierly dismisses "the fourth segment as of no metaphysical importance" he ignores the inconvenient circumstance that the ratio $d : a$, albeit doubtless small, is nevertheless not zero.[49]

Yet why should Plato hold that $a = b^2 = c^2$? Why should the illumination of Reason so greatly amplify descriptive deliverances of qualitative perception and quantitative conception?

Presumably the insight here is that we do not really understand something until we have embedded it within a larger framework of "scientifically" organized

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[47] See Benjamin Jowett, *Plato's Republic: Text, Translation and Commentary*, 3 vol.'s (Oxford: Clarendon Press, 1894), and Henry Sidgwick, "On a Passage in Plato's Republic, B. VI," *The Journal of Philology*, 2 (1869): 96-103.

[48] Henry Jackson, "On Plato's Republic VI, 509D sqq," *The Journal of Philosophy* 10 (1882): 132-150.

[49] Sidgwick, "On a Passage," 102.

systematization – that is until we comprehend its place and role in the larger scheme of things and are able to characterize it descriptively but to *explain* it "scientifically." Only when we know how an item figures in the larger explanatory framework of environing fact do we really understand it. Our comprehension of things is not *real* knowledge until we understand them in their wider systemic context. In short, what Plato seems to have in view is that higher kind of knowledge which Spinoza characterized as "an adequate knowledge of the essence of things" (*adaequata cognitio essentiae rerum*). In sum, intellect – that topmost "scientifically informed cognition" if you will – vastly amplifies the illumination provided by sensory information. In this perspective, Divided Line marks Plato as a quintessential rationalist.

Still, why should it be that a = b^2 = c^2 (with length measured in d units). Why is it a matter of *squaring* – why not have a come to c^3 or $1000c$?

Dialectics, so Plato tells us, calls for "a synoptic survey (*sunopsis*) of facts studied in the special sciences then relationships to one another and to the nature of things" (537C). And so this square root relationship should really not be seen as all that puzzling. After all, when n items are at issue there will be n stories to be told by way of individual description. But if systemic understanding demands grasping *how these items are related to one another*, then there will be $n \times n = n^2$ stories at issue.

And there is a further interesting aspect to the issue. This turns on a striking parallelism to a relationship in the modern theory of information known as Rousseau's Law[50] which maintains that the sort of cognitively significant amount of higher-level *knowledge* (K) provided by a body of raw *information* (I) stands merely at the square root of this body's size:

$$K = \sqrt{I} \quad \text{or} \quad K^2 = I$$

Such comparisons do of course involve something of a coincidence, very different sorts of considerations being at work in those two ranges of discussion. But all the same, there seems to be a commonality of perspective, rooted in the idea that there is a vast gap between the cognitive significance – the "illumination" provided by raw empirical information and that provided by a scientifically based systematization – constitutes a disparity to which a square-root relationship gives a seemingly natural mathematical embodiment.

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[50] On Rousseau's Law see Nicholas Rescher, *Epistemology* (Pittsburgh: University of Pittsburgh Press, 2004).

8. The Allegory of the Cave (514A-521B)

How is one to fit the Divided Line narrative of *Republic* VI into the wider framework of Plato's epistemology, and, in particular, to coordinate it with the Cave Allegory of *Republic* VII? Especially because The Cave Allegory lacks the formalization of the Divided Line narrative, commentators have expended much ink on the question of how the two are related.[51]

It is clear that the Cave story envisions three regions: (1) the cave wall with its fire-projected shadows and images, (2) the entire cavern with its fire-illuminated visible objects, and (3) the exterior with its sun-illuminated realities assembled to the Platonic Ideas. As regards these, a fundamental proportionality is contemplated, for Plato tells us that "The realm of sight is like the habitation in prison [i.e., the cave], and the firelight there is like the sunlight" (517B). So overall, we are presented with the dual proportionality:

the Good : ideas :: the sun : worldly things :: worldly things : images

Interestingly, Plato begs off (at 506b-507a) from dealing with the Good as such in favor of dealing with its "offspring." In effect, he says "Don't ask me what the Good *is*, ask me rather about what it *does*." (William James would love that!) And his response is that what the Good does is to serve a dual role. For one must distinguish between two questions "What is it that makes *p* be the case?" and "What is it that makes us think that *p* is the case?" – that is we can ask both about the ontological truth makers and the epistemological truth-markers that render the truth cognitively accessible. And Plato's line here is that as far as the world's facts are concerned one and the same potency plays both roles. For the idea of the Good is the basis both of the world's realities and of their knowability. As N. P. White concisely puts it, the idea of the Good is regarded by Plato "as the cause both of the being of intelligible objects as well as of our knowledge of them!"[52] Like the sun which enables living creatures both to exist and to be seen, the Idea of the Good is the basic source both of the knowable and of its knowability.

There is nothing all that strange about the fundamental idea of the Cave Allegory. The Platonic parallelism between eyesight and insight, between vision and understanding, between the light of the sun and the enlightenment of thought,

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[51] Virtually every commentary cited in our bibliography has much to say on the subject.

[52] Nicholas P. White, *A Companion to Plato's Republic* (Indianapolis.: Hackett, 1979), 180. It is striking, but not untypical, that White's commentary leaves the mathematical proportionalities of the Divided Line out of consideration.

is actually pretty much taken for granted by everyone. The student who grasps a mathematical concept immediately says "Now I see it." We say that it was a "flash of insight" that led Archimedes to exclaim *eureka*! In everyday use, "illumination" is as much mental as visual.

In the Cave Allegory, three relationships are thus analogized in terms of proportionality among three triads:[53]

The Good//Rational Insight//Ideas

The Sun//Sight//Visible Objects

The Fire//Supposition//Shadows & Images

The guiding idea is that the light of the fire in relation to the objects of the cave is like the illustration of the sun in relation to the objects outside. And so The Cave narrative envisions the analogy:

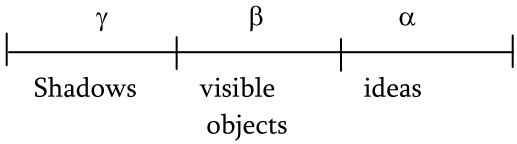
Shadows : objects : : object: ideas

But if we now adopt a mathematical perspective and shift from analogy to proportion some basic facts come more sharply into view. For looking at the situation in terms of a linear arrangement (something that the Cave Allegory invites but does not explicitly state) we would have:

Display 5
POSSIBLE CAVE-TO-LINE CORRESPONDENCES

Cave *Redistributional Match-Up of line Segments*

	I	II	III	IV
α	*a*	*a*	*a* + *b*	*a*
β	*b*	*b* + *c*	*c*	*c*
γ	*c* + *d*	*d*	*d*	*d*



[53] Scholars have disputed about just how many epistemic division are in play with the Cave Allegory. See, for example, Robinson, *Earlier Dialectic*, and John Malcolm, "The Line and the Cave," *Phronesis*, 7 (1962): 38-45. The tripartite picture contemplated here looks to be not only the simplest but also the most natural reading of the text.

with the result of the following proportionality:

$$\frac{\gamma}{\beta} = \frac{\beta}{\alpha} \text{ or equivalently } \alpha\gamma = \beta^2$$

This relationship is, in effect, simply the well-known geometric section of a harmonic mean amply elaborated upon in antiquity in the treatises of Niomachus and of Pappus.[54] So if we once more conduct our mensuration in terms of γ-units, (so that $\gamma = 1$), then we again have $\beta = \sqrt{\alpha}$. Mathematical proportionalities once more confront us.

But now there arises the critical question of bringing books VI and VII of the *Republic* into unison is this: Can the Divided Line narrative be reconciled with the Cave Allegory? A good deal of ink has been spilt over the question of whether the Cave and the Line account can be resolved. Robinson 1952 maintains that Plato's characterization of the Cave situation "forbids us to put it in exact correspondence with his Line," but other commentators have disagreed: for example, Gould,[55] Malcolm[56] and Morrison[57].

One potential strategy of reconciliation would proceed by reconfiguring the line segments to achieve a correspondence. The possibilities available

Display 6
FOUR COGNITIVE PERSPECTIVES UPON THREE
SORTS FOR OBJECTS

Perception	SUPERFICIAL IMPRESSIONS	mere sense-impressions
	OBSERVATIONS	mundane realities
Cognition	GEO-METRICAL FACTS	
	IDEALIZATIONS	pure theorizing

here are inventoried in Display 6. As just noted, the Cave Allegory requires that:

[54] See T. L. Heath, *A History of Greek Mathematics*, Vol. 1 (Oxford: Clarendon Press, 1921), 87.
[55] Gould , *Development.*
[56] Malcolm, "The Line," 38-45.
[57] Morrison, "Two Unresolved," 212-31.

$$\frac{\gamma}{\beta} = \frac{\beta}{\alpha} \text{ or equivalently } \alpha\gamma = \beta^2$$

However, the Divided Line narrative requires both that $b = c$, and further that $a = b^2$ when d is 1. So in relation to those Display 6 cases we now re- quire the following equations for transposing the α-to-γ range into the a-to-d range:

Case I. $b^2(b + 1) = b^2$. Not possible unless $b = -1$ or $b = 0$.

Case II. $b^2 = (2b)^2$. Not possible unless $b = 0$.

Case III. $b^2 + b = b^2$. Not possible unless $b = 0$.

Case IV. $a = c^2$. No problem! [See section 7 above!]

And so unless $b = 0$, the only viable match-up between Line and Cave is represented by IV which, exactly as one would surmise, leaves the Divided Line mathematicals (here represented by b) entirely out of view. We thus have a choice: we can annihilate the mathematical or simply let them drop out of sight. For on such an approach, *dionoia* with its concern for *mathematika* is dismissed. It simply appears to have vanished from the Cave account.[58] How can this be?

It would be tempting to try to reconcile the Line and the Cave accounts by the speculation that what is at issue is a matter of four cognitive perspectives upon three sorts of objects – rather along the lines of Display 6. Substantially this approach to the matter is taken in Wieland 1982. As he sees it, the things at issue with b and c the sensibles and the arithmeticals represent one selfsame group of items, the natural world's concreta, viewed two different prospectives, one qualitatively as objects of perception (*Gegenstände der Wahrnehmung*) and once quantitatively as material representations of forms (*Abbilder* [*der Ideen*]). Since merely different dispositions (*Einstellungen*) toward the same objects are at issue, the two representing segments should be equal. This all sounds plausible enough, but as the analysis relating to Display 5 clearly shows, it just does not square with the treatment of the mathematicals in the Divided Line narrative. For the preceding analysis blocks this otherwise attractive prospect of amalgamating b and c. While the overall account insists on their being co-equal, it blocks the prospect of their fusion via the impracticability of alternative II above.

Again, it might be tempting to conjoin *dianoia* and *epistêmê*, then consolidating the two higher cognitive facilities into one. But the impracticability of alternative III rules this out.

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58 That those accounts are irreconcilable is sometimes maintained. (See, for example, Robinson, *Earlier Dialectic*, 181-82.) However the simplicity of the reason for this – viz., the Cave's indifferent to *dionoia* – has not been duly emphasized.

By a bit or fanciful legerdemain I.M. Crombie revamps the Cave account into four "stages" of image-to-original relationship in a way that coordinates the Cave with the Line, claiming that "Plato intended us to suppose that in the parable of the Cave he was putting flesh upon the bones of the skeleton he set out in the Line."[59] In effect he resorts to the match-up:

a	α
b	β (part)
c	β (part)
d	γ

This seemingly provides for a smooth coordination between the Line and the Cave representations. But when we look at the matter in the reverse direction (from Cave to Line) we return to Case II of Display 5 and fall back into unavoidability.

Rosemary Desjardines goes yet further in having it that $a = c + d$.[60] Not only does this fail to be endorsed by Plato,[61] but in the context of the ratios that he explicitly specified it would have some strange consequences. For example, since

$$\frac{a}{b} = \frac{a+b}{c+d}$$

it would mean that

$$a^2 = b(a + b)$$

Note now that if we constructed out measurements in terms of *a* as a unit ($a = 1$), then the ratio d : c : b : a would be .38, .62, .62, 1.0 which would be bizarre given the intended interpretation of the Line.

W.D. Ross believes that further ratios are also needed "and that this is mathematically impossible is only an indication of the fact that the line, being a symbol, is inadequate to the whole truth which Plato wanted to symbolize."[62] And some commentators incline to think that the mathematician's symbols are somehow akin to shadows of the Cave allegory.[63] (After all, both leave substance and content aside and deal only with structure and thus suggests a coordination of

[59] I. M. Crombie, *An Examination of Plato's Doctrines, Vol. I: Plato on Man and Society* (London: Routledge & Kegan Paul, 1962), 116.
[60] See Desjardines, *Rational Enterprise*, 491.
[61] See Pritchard, *Plato's Philosophy*, 97-98.
[62] Ross, *Plato's Theory*, 45-46.
[63] Ferguson, "Plato's Simile," 148.

dianoia and *eikasia*.) But once again, the requisite detail for such an approach simply cannot be implemented satisfactorily.

Colin Strang[64] analyzes the Cave/Line relationship in a somewhat eccentric manner. With respect to the Line, he views it as having *five* divisions:

$\left.\begin{array}{l}\text{noêsis,}\\ \text{dianoia}\end{array}\right\}\text{epistêmê}$

doxa

pistis

eikasia

This introduces *doxa* as a separate division on a level with the other standard four. And as regards the Cave, he sees *all* five of these as functioning outside (i.e., above and beyond) the Cave and its firelight in the outer realm of the sun. It is unclear, however, both how this makes good textual sense, let alone how it provides for a more cogent philosophical systematization. Accordingly, Strang is constrained to insist that Plato's own contention involves a variety of "misdirections," and maintains that "No interpretation... can hope to emerge unscathed from the text," claiming that his account "makes better philosophical sense than its rivals and can more easily explain away the anomalies that remain." The first part of this contention may well be true, but the second part looks to be adrift in a sea of troubles. In particular, Strang's account simply ignores the whole manifold of mathematical proportionalities that lay the groundwork for the Divided Line.

Plato himself was keenly aware that the Divided Line narrative leaves a great deal unsaid and that its adequate exposition would require a much further explanation. In Book VII of the *Republic*, he has Socrates say: "But let us not, dear Glaucon, go further into the proposition between the lines representing the opinionable (*doxaston*) and the intelligible (*noêton*) so as not to involve ourselves in any more discussions than we have had already" (534A) Here Plato is clearly not *retracting* the Divided Line narrative but simply noting that it need not be further elaborated for the limited purposes just then at hand. And as Wedberg rightly observes "it is merely about the object of mathematics that [further] information is being withheld" at this particular juncture.[65]

So, what is one to make of this? How can one possibly account for the disappearance of the mathematics-oriented *noêsis* of segment *b* in the transit from

[64] Colin Strang, "Plato's Analogy of the Cave," *Oxford Studies in Ancient Philosophy*, IV (1986): 19-34.

[65] See Wedberg, *Plato's Philosophy*.

the Line to the Cave account? There is, it would seem, one plausible way to do it – one that involves a fundamental shift of perspective.

Suppose that *dianoia* is not conjoined with *epistêmê*, and *b* combined with *a* as per Case III of Table 6, but rather it is that the operations of *dionoia* are folded into and absorbed into *epistêmê*, high-level so that *b* effectively vanishes and its function is now provided for from within. The point is that even if we refrain from seeing the objects of mathematizing *dionoa* as themselves being Ideas (or Form), nevertheless as abstract and unchanging realities, they will fall into the same generically sense-transcending domain. Mathematics is thus seen as one integral part of a complex effort to detach people from the realm of sense: "to turn the soul's attention upwards from the sensible to the intelligible" as one recent commenter puts it.[66] To achieve real understanding we must leave any and all experientially guided suppositions behind, abandoning mere hypotheses for the solid ground of rationally apprehended principles.

Mathematics itself thus becomes transformed into a science not just of basic ratiocination (which must inevitably proceed from premisses) but one of rational insight because the fully trained mathematician comes to see why those hypotheses (those four fundamental definitions, axioms, and possibilities) come to be just what they are.[67] Mathematics is now effectively seen as part of dialectic and mathematical training becomes an integral component of the *paidea* of the philosopher-king. That is, we effectively shift from *ousia* to *paideia*, from ontology to taxonomy. On this basis, mathematics is no longer to be seen as a distinct discipline with a subject-matter realm of its own (the *mathematica*), but rather a methodology of thought-descriptive that is an essential part of the training of the philosopher-kings.[68] Mathematics is thus cast in the role of the training-ground for abstracting from the mundane details of the sensible world and ratiocination (*dianoia*) is comprehended *within* reason (*epistêmê*) and *b* is not a supplement to *a* but a component part of it. In this regard, the present analysis gives full marks to Henry Jackson, who wrote well over a century ago:

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[66] Ian Meuller, "Mathematical Method and Philosophical Truth," in *The Cambridge Companion to Plato*, ed. Richard Kraut (Cambridge: Cambridge University Press, 1992), 189. Here one need not go quite as far as Erich Frank and hold that "Plato die Idea noch rein quantitativ als die blosse mathemeatische ideale Form der Dinge, d. h. als Zahl gefasst hat" (Frank, *Pythagoraer*, 60).

[67] Compare Myles F. Burnyeat, "Platonism and Mathematics: A Prelude to Discussion," in *Mathematics and Metaphysics in Aristotle*, ed. Andreas Graeser (Bern & Stuttgart: Paul Haupt, 1987) and Burnyeat, "Plato", as well as Gill, "Plato" and Gill, "The Good".

[68] On this issue see Ian Robins, "Mathematics and the Conversion of Mind: *Republic* VII 522C1-531E3," *Ancient Philosophy*, 15 (1995): 359-91.

> There is no place for the mathematika [in the cave account]. Plato, as I understand him, is here concerned not with mathematika as apposed to other noêta, but with mathematika as types of noêta.[69]

From this standpoint, the condition of *dianoia* (*b*) is like that of a conquered state that is neither annihilated by nor annexed to another, but rather bodily absorbed into it. So regarded mathematics acquires a different status, not as a distinctive field of inquiry but as a characteristic methodology of thought – a circumstance that merits the substantial emphasis that it in the educational deliberation of Book VII. But something comparatively radical is clearly needed.

No wonder, then, that with such a shift of perspective considerable confusion might arise. In some of the expositions of Books VI and VII of the *Republic* – Fine 1990 for example – the mathematical aspect of the Divided Line is a non-entity, with the detail of those proportions seen as philosophically irrelevant. Other accounts take note of such detail as the fact that $c = d$ but do not venture into an explanatory rationale. (Fogelin 1972, for example.) But be this as it may, it should be stressed that no interpretation of Books VI and VII of the *Republic* deserves to be deemed adequate that does not integrate the philosophical views being articulated with the mathematical detail being used in their articulation. And, above all, the Whewell Relations cannot simply be dropped from view.

Granted, the proportionalistic structure of the Divided Line, which, after all, is its very reason for being as such, is something that simply does not interest various commentators. No doubt, the tentative suggestions of the present discussion can and should be improved upon. But the overall project of getting this sort of thing right would seem to be something from which Plato's interpreters cannot consciously beg off, and the offhand dismissal of the whole project by various interpreters is something that does little credit to Platonic scholarship. A couple of generations ago, A. S. Ferguson wrote that: "The similes of the Sun, Line, and cave in the *Republic* remain a reproach to Platonic scholarship because there is not agreement about them."[70] This may be going a bit too far. It is simply too much to expect scholars to agree on *what Plato meant*. But *that he meant something* – and something sensible at that – ought not be to a bone of contention.[71, 72]

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[69] Henry Jackson, "On Plato," 141.

[70] Ferguson, "Plato's Simile," 190.

[71] I cannot forego the observation that with regard to the specific issue being investigated here – namely the proportionalities at work in the Divided Line narrative of *Republic* VI and the Cave Allegory of *Republic* VII – I find the 19th century Platonic commentators – and in specific Whewell, Jackson, and Ferguson – more helpful than their 20th century successors.

[72] I am grateful to Paul Scade for his constructive comments.

Appendix 1

PROOF THAT $b = c$

$$\frac{a}{b} = \frac{c}{d} = \frac{a+b}{c+d}$$

is given as basis

Now observe:

$$b = \frac{a(c+d)}{a+b} = \frac{ac+ad}{a+b} = \frac{ac+bc}{a+b} = \frac{c(a+b)}{a+b} = c$$

Note: It further follows that we shall also have:

$$\frac{a+b}{c+d} = \frac{a+c}{b+d} = \frac{a+b}{b+d}$$

Appendix 2

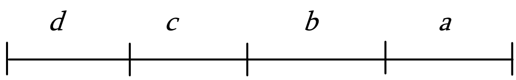
PROOF THAT $a = b^2 = c^2$ WHEN d IS THE UNIT OF MEASUREMENT

Let the ratio of c to d be r, so that $c = dr$. Then since $b = c$, we also have $b = dr$. But since the ratio of a to b is also to be r, we have $a = rb = dr^2$. Thus the quartet d, c, b, a will be d, dr, dr, dr^2. This is exactly what Whewell noted in *The Philosophy of Discovery* – and so represents what may be referred to as the Whewell Relations. Accordingly, if we employ d as the unit of measure, so that $d = 1$, then this quartet will be 1, r, r, r^2. On this basis $a = b^2 = c^2$ in the special case when $d = 1$, thought in general we shall simply have $a = b^2/d = c^2/d$.

Appendix 3

OBSERVATIONS ON THE PLATONIC SECTION

The abstract proportionalities at issue with Plato's Divided Line will not of themselves determine the relative size or magnitudes of the quantities involved. For consider once more those proportions superimposed upon the linear scheme



namely

$$\frac{a}{b} = \frac{c}{d} = \frac{a+b}{c+d}$$

Here *a* could in theory be as small as *d* itself, seeing that $a = b = c = d = 1$ will satisfy all of these proportionalities. On the other hand, *a* can be larger than the rest by any desired quantity whatsoever. For let us once more proceed to measure length in terms of *d*-units (i.e., $d = 1$). Then a discrepancy in the size in the magnitudes at issue of any size whatsoever will be able to satisfy all those proportionalities – however large is may be – as long as:

$$a = k$$
$$b = \sqrt{k}$$
$$c = \sqrt{k}$$
$$d = 1$$

with the magnitude of *k* left open. So as regards the potential disparity of *c* and *a*, the sky is the limit.

An interesting perspective emerges when measurement is made in terms of *b* ($b = 1$). For *d*, *c*, *b*, *a* will now stand as *d*, 1, 1, $\frac{1}{d}$. For since $b = c = 1$, a relationship of reciprocal complementarity between *d* and $a = \frac{1}{d}$ must obtain. That is, with *d* a very small quantity *a* will be a very big one (and conversely). The dimness of those mundane reflections is in diametrical contrast with the brightness of sunshine.